CUA

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Ph.D. Comprehensive Examination
Physics Department
Spring 2017
Thursday, March 30, 2017 and Friday, March 31, 2017
Room 136 – Hannan Hall

GENERAL INSTRUCTIONS:

This examination is divided into four sections as follows:

<u>Thursday, March 30th</u>

9:00am – 12:00pm	Classical Mechanics – 2 Questions
1:00pm – 5:00pm	Electricity & Magnetism – 3 questions

<u>Friday, October 24, 2014</u>

9:00am – 12:00pm	Statistical Mechanics – 2 Questions

In each of the four subject areas, you may answer the 500-level question in place of the 600-level question

Directions for Blue Books:

1) **DO EACH PROBLEM IN A SEPARATE BLUE BOOK**
2) **PUT YOUR NAME ON EACH BOOK**
3) **LABEL EACH BOOK WITH THE CORRECT PROBLEM NUMBER**

You may user:

1) **Standard Calculator (Non-Programmable)**
2) **Copy of Schaum's Outlines**
3) **Scratch Paper**

EXAMPLE: "Mechanics 600-1"

A long cylindrical conductor of radius a, has an off-center cylindrical hole of radius a/2 down its full length, as shown in the figure below.

a. If a current I flows through the conductor into the page, then what is the strength and direction of the magnetic field in A?

b. If a current I flows through the conductor into the page, then what is the strength and direction of the magnetic field in B?

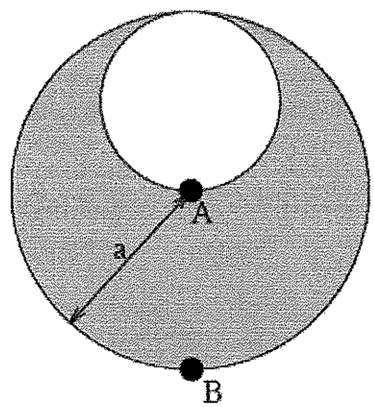


Figure 1: Cross section of a long cylindrical conductor with an off-center cylindrical hole.

A line charge with linear charge density τ is placed parallel to, and a distance R away from, the axis of a conducting cylinder of radius b held at a fixed voltage such that the potential vanishes at infinity. Find

a) the magnitude and positions of the image charge(s);

b) the potential at any point (expressed in polar coordinates with the origin at the axis of the cylinder and the direction from the origin to the line charge as the x axis), including the asymptotic form far from the cylinder;

c) the induced surface-charge density, and plot it as a function of angle for R/b =2, 4, in units of $\tau/2\pi b$;

d) the force on the charge.

A region in space contains a total positive charge Q that is distributed spherically such that the volume charge density $\rho(r)$ is given by:

$\rho(r) = \alpha$ for $r \le R/2$

$\rho(r) = 2\alpha(1-r/R)$ for $R/2 \le r \le R$

$\rho(r) = 0$ for $r \ge R$

where α is a positive constant having units of C/m^3.

a). Determine α in terms of Q and R.

b). Derive an expression for the magnitude of the electric field $\vec{E}(r)$ for $r \le R/2$.

c). Derive an expression for the magnitude of $\vec{E}(r)$ for $R/2 \le r \le R$.

d). Derive an expression for the magnitude of $\vec{E}(r)$ for $r \ge R$.

e). What fraction of the total charge is contained within the region $r \le R/2$?

f). If an electron with charge $q'=-e$ and mass m is oscillating back and forth about r=0 (the center of the distribution) with an amplitude less than R/2, describe <u>qualitatively</u> the nature of the oscillation.

g). What is the period of the motion in part f)?

h). If the amplitude of the motion described in part f) is greater than $R/2$, describe <u>qualitatively</u> the nature of the oscillation.

A radiating quadrupole consists of a square of side a, with charges +/-q at alternate corners. The square rotates with an angular velocity ω about an axis normal to the plane of the square and through its center. Calculate (all in the long-wavelength approximation). NOTE: you will want to use Cartesian coordinates.

a) the charge distribution, $\rho(\mathbf{x},t)$. Note, this will include delta functions.

b) the quadrupole moments. The general definition of the quadrupole moments is:

$$Q_{\alpha\beta} = \int (3x_\alpha x_\beta - r^2\delta_{\alpha\beta})\rho(\mathbf{x},t)\, d^3x$$

and you need Jackson's quadrupole vector: $Q_\alpha = \sum_\beta Q_{\alpha\beta}\, n_\beta$

where the n's are unit vectors along the x- and y- axes.

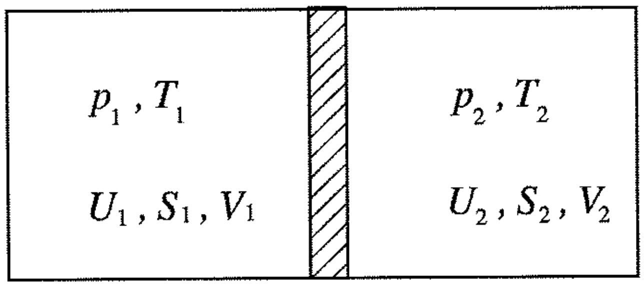
c) the electric and magnetic fields (with their angular dependence); you can start with the magnetic field:

$$\mathbf{B} = -\,[(ik^3 e^{ikr})/(6r)]\ \mathbf{n}\ \mathbf{x}\,\mathbf{Q}(\mathbf{n})$$

d) the time averaged power per unit solid angle, noting

$$dP/d\Omega = (c/288\pi)\, k^6\, |[\mathbf{n}\ \mathbf{x}\,\mathbf{Q}(\mathbf{n})]\ \mathbf{x}\,\mathbf{n}\,|^2$$

e) the total radiated power. ($d\Omega = 2\pi \sin(\theta)d\theta$)



An adiabatically insulated system with constant volume V is divided into two parts at different pressures and temperatures. Show that after reaching the thermodynamic equilibrium temperature and pressure of the two parts of the system are aligned.

Take advantage of the condition of internal energy minimum: $\delta U > 0$

(a) For the canonical ensemble, show that the specific heat is proportional to the variance of the energy, i.e.:

$$C_v = \left(\frac{\partial\langle E\rangle}{\partial T}\right)_V \propto \langle E^2\rangle - \langle E\rangle^2,$$

and find the constant of proportionality.

Note that

$$C_v = \left(\frac{\partial\langle E\rangle}{\partial T}\right)_V.$$

(b) For the grand canonical ensemble, show that

$$\langle E\rangle = \mu\langle N\rangle - \frac{\partial\, lnZ_G}{\partial\beta}.$$

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Spring 2017
Statistical Mechanics SM 600-2

For the non-interacting quantum gas in three dimensions with energy spectrum

$$\epsilon(\vec{p}) = \frac{|\vec{p}|^2}{2m}$$

show, using integration by parts, that

$$PV = \gamma \langle E \rangle,$$

and find γ. Show this separately for both Fermions and Bosons. You'll find that the value of γ is the same in both cases.

This result does not depend on the particle degeneracy *M*.

The following formulas for the non-interacting quantum gas may be useful:

$$lnZ_G = \beta PV = \mp \frac{MV}{h^3} \iiint_{-\infty}^{\infty} d^3p \; ln[1 \mp exp(-\beta\epsilon(\vec{p}) + \beta\mu)]$$

$$\langle E \rangle = \frac{MV}{h^3} \iiint_{-\infty}^{\infty} d^3p \frac{\epsilon(\vec{p})}{1 \mp exp(-\beta[\epsilon(\vec{p}) + \mu])}$$

Here, the top/bottom signs refer to Bosons/Fermions respectively, and *M* is the particle degeneracy, i.e. the number of different particle "flavors". Also, each integral in the triple integrals runs from $-\infty$ to ∞.

A particle of mass m moving in an inverse square law force field, $-\mu m/r^2$ such as a small planet or asteroid in the gravitational field of a star or larger planet. The potential function satisfies:

$$-\frac{\partial V}{\partial r} = -\frac{\mu m}{r^2}$$

a) What is the potential *V(r)*?

b) Write the expression for kinetic energy in plane polar coordinates r(t),θ(t)

c) Write expression for Lagrangian

d) Write equation of motion

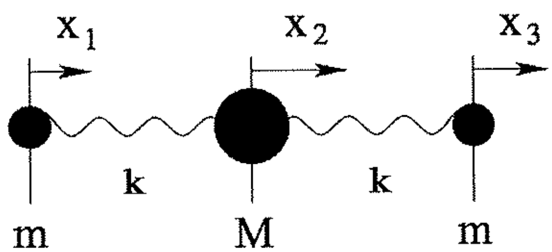

Figure 1: Tri-atomic system (for instance CO_2: O – C – O).

Consider a system of three particles connected via two springs (see the figure). Two particles have identical mass m and one, the center particle, have mass M. The particles are connected with two identical springs, with spring constants k. If we only consider movements in one plane, that is, no rotation or bending, find:

a) Find the Lagrangian that describes the movement of the three masses in the x–direction.

b) Find the equation of motion for each of the masses.

c) The matrices describing the kinetic and potential energies for small oscillations of this three-particle system are

$$\mathsf{T} = \begin{pmatrix} m & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & m \end{pmatrix}$$

$$\mathsf{V} = \begin{pmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{pmatrix}$$

Find the eigenvalues λ_i and the corresponding un-normalized eigenvectors for $|\mathsf{V} - \lambda \mathsf{T}| = 0$.

d) Each of the eigenvectors describe the relative motion of the three particles. Interpret your results from c) in terms the relative motion and describe in words, or in a figure, how the particles move. These are the normal modes of this system.

PhD Comprehensive Examination
Spring 2017
Classical Mechanics 600–2

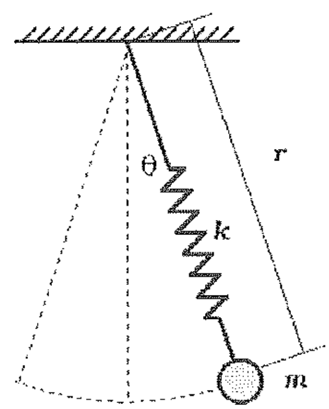

Figure 1: Spring pendulum

A spring pendulum consists of a mass m attached to one end of a massless spring with spring constant k. The other end of the spring is attached to a fixed support. With no weight on the spring, its length is l. Assume that the motion is fixed to a vertical plane.

a) Derive the Lagrangian for the system when a weight of mass m is attached to the spring.

b) Derive the equations of motion for this system.

c) Assume small angular and radial displacements from equilibrium and solve the equations of motions.