CUA

THE CATHOLIC UNIVERSITY OF AMERICA

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MS Comprehensive Examination
Physics Department

Fall 2008

Thursday, October 23 and Friday, October 24, 2008

Room 133 - Hannan Hall

GENERAL INSTRUCTIONS:

This examination is divided into four sections as follows:

Thursday, October 23, 2008
9:00 a.m. - 12:00 Noon Classical Mechanics - 2 questions

1:00 P.M. - 5:00 P.M. E & M - 2 questions

Friday, October 24, 2008
9:00 a.m. - 12:00 Noon Thermodynamics/Stat. Mech. - 2 questions

1:00 P.M. - 5:00 P.M. Modern Physics/Quantum Mech. - 2 questions

DO EACH PROBLEM IN A SEPARATE BLUE BOOK

PUT YOUR NAME ON EACH BOOK

LABEL EACH BOOK WITH CORRECT PROBLEM NUMBERS
FOR EXAMPLE: Mechanics 600-1 or Electromagnetism 500-2

YOU MUST DO TWO (AND ONLY TWO) OF THE QUESTIONS IN EACH AREA.

OF THE TOTAL OF EIGHT QUESTIONS THAT ARE DONE, A MINIMUM OF TWO MUST BE FROM THE 600-LEVEL QUESTIONS.

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RULES FOR THE PHYSICS COMPREHENSIVE EXAMINATION

October 23 & 24, 2008

In order to assure an equitable and fair Comprehensive Examination for all students the following rules will apply to the examination administered by the Department of Physics.

1. The examination will be closed book. Students will not be permitted to bring to the examination room materials such as notes, reference books, calculators, or other aids of any form. The Physics Department will supply copies of the *CRC Mathematical Handbook*, Schaum's *Mathematical Handbook*, *Table of Functions* by Jahnke and Emde, and the NBS *Handbook of Mathematical Functions*, for the use of those taking the exam should they feel the need for these references during the examination.

   The Physics Department will supply calculators for use during the examination.

   Students who want to use their own mathematical tables and/or handbooks must get permission from the Chairman prior to the beginning of the comprehensive examination. These tables and/or handbooks will remain under the control of the department until after the examination is completed, but will be available to their owners during the exam.

2. If needed, students will be allowed a short break, not to exceed ten minutes, during the examination period. While absent from the examination room students may not consult material or other sources of information concerning any matter pertaining to the examination. Students who leave the examination room will turn in all of their examination materials to the proctor before leaving the room. Examination papers will be returned to the students when they reenter the examination room. Students who fail to return within ten minutes will be considered to have completed that portion of the examination.

   Only one student will be permitted to leave the examination room at a time.

A moon of mass m orbits with angular velocity ω around a planet of mass M. Assume the moon is a point-like particle and that $m \ll M$. However, the planet spins about its axis with angular velocity Ω and its spin axis is perpendicular to the plane of the moon's orbit. Let I be the moment of inertia of the planet about its axis and r be the distance of the moon to the center of the planet. You can consider the planet to be fixed in space.

(a) Find an expression for the total angular momentum L of the system about its center of mass.

(b) What is the total energy E?

(c) Eliminate r from the expressions obtained in (a) and (b) by applying Newton's law on circular motion. Show that $\omega=\Omega$ leads to an extremum of the energy. Under what conditions is the energy at a stable minimum?

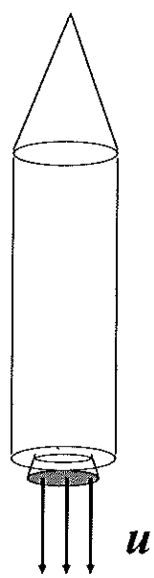
Consider an amateur rocket, initially at rest, that is launched from the earth on a vertical trajectory. The rocket experiences no atmospheric friction, and is under the influence of gravity, where the constant acceleration due to gravity is expressed as g. The rocket has a constant exhaust velocity u, and a constant mass ejection rate k, in mass per second. The initial fueled rocket has a mass of m_0, and the mass of the rocket exhausted of fuel, at time of burnout is m_f.

In terms of u, k, m_0, m_f, and g derive an expression for:

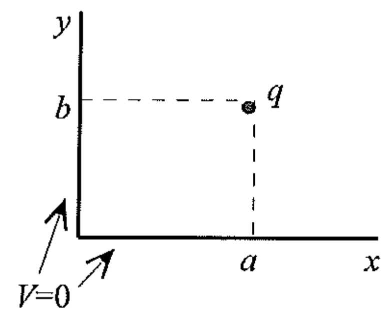
a.) Total time elapsed time from lift-off to burn-out,

b.) Vertical velocity of the rocket at time of burnout,

c.) Altitude of the rocket at the time of burnout.



Two semi-infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge q at position (a,b).



(a) Set up the image configuration and calculate the potential in this region.

(b) What is the force on q? How much work does it take to bring q in from infinity?

(c) Suppose the planes met at some angle other than 90°; would you still be able to solve the problem by the method of images? If not, for what particular angles does the method work?

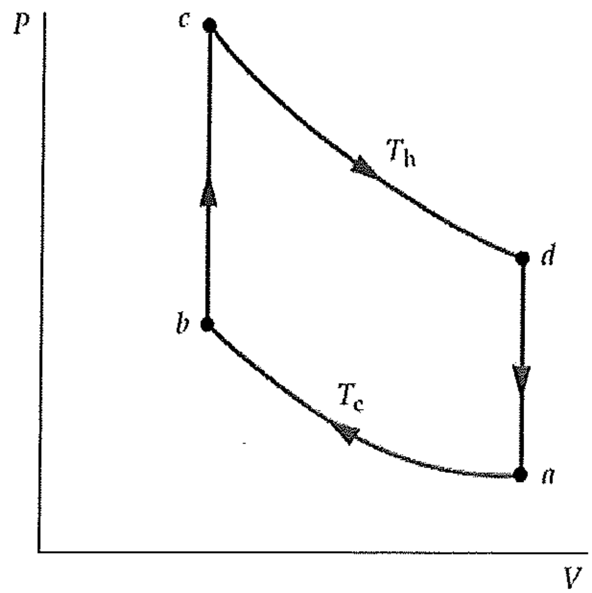
A metal sphere of radius a carries a charge Q. It is surrounded by a shell of thickness d consisting of dielectric material of permittivity ϵ.

(a) Find the potential at the center (relative to infinity).

(b) Compute the polarization and bound charge in the dielectric.

(c) Determine the surface charge density at the outer and inner surface of the dielectric shell.

In the Stirling cycle shown in the figure process ***ab*** is an isothermal compression, process ***bc*** is heating at a constant volume, process ***cd*** is an isothermal expansion, and process ***da*** is cooling at constant volume.



a) Find the efficiency of the Stirling cycle in terms of the temperatures T_h and T_c and the volumes V_a and V_b.

b) Compare the efficiency of the Stirling cycle and the Carnot engine operating between the same maximum and minimum temperatures.

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MS Comprehensive Examination
Fall 2008
Thermodynamics 500-2

In a Joule-Thompson or "throttling" process of gas liquification, temperature reduction is attained through a constant enthalpy transformation from an initial state i, with temperature T_i and pressure P_i, to a final state f, with temperature $T_f < T_i$ and pressure $P_f < P_i$.

(a) Express the Joule-Thompson coefficient $\left(\partial T/\partial P\right)_H$ in terms of $\alpha = \frac{1}{V}\left(\partial V/\partial T\right)_P$ and $C_P = \left(\delta Q/\delta T\right)_P$. The enthalpy is $H = U + PV$ where U is the internal energy.

(b) Determine $\left(\partial T/\partial P\right)_H$ and $\alpha = \frac{1}{V}\left(\partial V/\partial T\right)_P$ specifically for an ideal gas. Could an ideal gas be liquified by such a "throttling" process?

a) Write the possible wave functions $\psi_1(x)$, $\psi_2(x)$, ... for a particle of mass m confined to a one-dimensional "box" (infinite square well) of length L, extending from $x=0$ to $x=L$:

$$V(x) = 0 \quad 0 \le x \le L$$
$$= \infty \quad x<0,\ x>L$$

Check that each function is normalized.

b) Find the energy of a particle in the ground state (described by wave function ψ_1) and in the first excited state (ψ_2).

c) Find the probability that the particle will be found in the "middle half" of the box, i.e. $\frac{L}{4} \le x \le \frac{3L}{4}$, for

i) the ground state wave function ψ_1

ii) the first excited state wave function ψ_2.

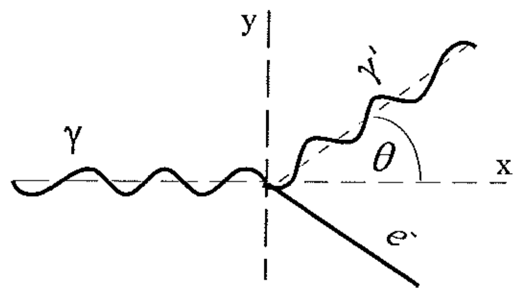
Sketch both wave functions, and verify that your calculations are sensible.

Note: Parts (d) and (e) do not depend on parts (b)–(c).

d) Write the wave function $\psi(x_1,x_2)$ for **two particles** of mass m in the box of length L, where x_1 and x_2 are the positions of the individual particles. Write your answer in terms of the single-particle functions ψ_n, assuming that one particle is in the ground state and the other particle is in the first excited state, and that the two particles are **distinguishable**.

e) Do the same as part (d) for the case that the two particles are **indistinguishable fermions** (e.g. two electrons), including the requirements of the Pauli exclusion principle.

Consider the collision of a photon of wavelength λ and 4-vector momentum $(E,\vec{p}) = (hc/\lambda, h/\lambda, 0, 0)$ with an electron at rest.



Use energy and momentum conservation to show that if the photon is scattered into the angle θ, the wavelength λ' of the scattered photon is given by $\lambda' - \lambda = \lambda_C(1-\cos\theta)$, where $\lambda_C = h/m_e c = 2.426 \cdot 10^{-9}$ m , h is Planck's constant and m_e the mass of the electron.

Foucault's pendulum is suspended from a point $(0,0,l)$ above ground on the northern hemisphere and brought to swing in a vertical plane with small amplitude.

(a) Let $(\hat{x},\hat{y},\hat{z})$ be the axes of a coordinate system associated to the earth with $\hat{z}$ directed upwards and $\hat{x}$ along a geodesics facing towards the equator. Obtain the equation of motion for this system.
Note: The tension on the thread is $\vec{F}_{tens} = |F_{tens}|(-{x}/{l}\,\hat{x} - {y}/{l}\,\hat{y} + (1 - {z}/{l})\hat{z})$

(b) Ignoring the centrifugal force and any velocity pointing outwards (to the sky) $0 \approx \dot{z} << \dot{x},\dot{y}$, solve the equation of motion and find the frequency of horizontal angular deflection ($\alpha = {d\varphi}/{dt}$) of the pendulum.

(c) Determine the horizontal projection of the path described by the pendulum and its velocity for the start conditions: $x(0) = a$; $y(0) = 0$; $\dot{x}(0) = \dot{y}(0) = 0$.
Determine the times and velocities when the pendulum arrives at turning points. Sketch the projection of the pendulum's path on the horizontal plane.

(d) Describe the motion of Foucault's pendulum if it is brought to swing by pushing the bob from the equilibrium point in the y-direction by an initial velocity $v_{0y} = a\sqrt{g/l} = a\omega_0$.
Determine the times and velocities when the pendulum arrives at turning points. Sketch the projection of the pendulum's path on the horizontal plane.

The Lagrangian for a system of one degree of freedom can be written as

$$L = \frac{m}{2}(\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2\omega^2) .$$

(a) What is the corresponding Hamiltonian? Is it conserved?

(b) Introduce a new coordinate defined by $Q = q \sin \omega t$. Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is H conserved?

Consider a homogeneous linear medium with constant permittivity ϵ, permeability μ and conductivity σ:

$$\boldsymbol{B} = \mu \boldsymbol{H}, \quad \boldsymbol{D} = \epsilon \boldsymbol{E}, \quad \boldsymbol{J}_{free} = \sigma \boldsymbol{E}.$$

Assume that the medium remains electrically neutral ($\rho_{free} = 0$), and that μ, ϵ, and σ are real.

a) Starting with Maxwell's equations, show that the electric field obeys the wave equation

$$\nabla^2 \boldsymbol{E} - \epsilon\mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \sigma\mu \frac{\partial \boldsymbol{E}}{\partial t} = 0,$$

and that the magnetic field obeys the same equation.
Hint: Use the vector identity $\nabla\times(\nabla\times\boldsymbol{F}) = \nabla(\nabla\cdot\boldsymbol{F}) - \nabla^2\boldsymbol{F}$.

Parts (b)-(f):

Consider a plane monochromatic wave in the z direction,

$$\boldsymbol{E}(z,t) = \hat{z}\, E_0\, e^{i(kz-\omega t)}$$

where E_0 is a complex constant. (The physical field is the real part of $\boldsymbol{E}$.)

b) Derive an expression for the (complex) wave number k in terms of μ, ϵ, σ and ω. Show that k *must* be complex when $\sigma \neq 0$.

c) Let the wave number be written $k = k_r + i k_i$ (where k_r and k_i are real numbers). Determine $k_r^2 - k_i^2$ and $k_r k_i$. You do not need to solve for k_r and k_i in general.

d) For a "good conductor" (when σ is sufficiently large – how large?), show that $k_i \approx k_r$, and find k_i and k_r in this approximation.

e) Show that the expression for the electric field can be written as the product of a sinusoidal wave and an attenuation factor. In the approximation of part (d), find the "skin depth" (characteristic penetration distance) for the wave in this medium. Does the skin depth increase or decrease with frequency?

f) In the approximation of part (d), find the **ratio of amplitudes** $\frac{|B_{max}|}{|E_{max}|}$ and the **phase difference** $\delta_B - \delta_E$ between the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ of the wave. (Hint: consider $\nabla\times\boldsymbol{E}$ for the plane wave given above.) What are the corresponding values for waves in free space?

A particle of mass m and charge q is restrained by an ideal spring (massless and of zero free length) with a spring constant η. A monochromatic wave with an electric field

$$\vec{\mathbf{E}}(x,t) = \vec{\mathbf{E}}_0 e^{i(kx-\omega t)}, \quad (k = \omega/c)$$

impinges on this system, with a wavelength much greater than any displacement of the particle.

(a) Find the angular distribution of the scattered power per unit solid angle.

Hint: In the radiation zone, the vector potential, the magnetic induction, the electric field, and the angular distribution of the radiated power of a harmonically time-varying dipole moment $\vec{\mathbf{p}}_0 e^{-i\omega t}$ are given by

$$\vec{\mathbf{A}} = -ik\vec{\mathbf{p}}_0 \frac{e^{i(kr-\omega t)}}{r}; \quad \vec{\mathbf{B}} = ik\hat{\mathbf{n}} \times \vec{\mathbf{A}}; \quad \vec{\mathbf{E}} = -\hat{\mathbf{n}} \times \vec{\mathbf{B}}; \quad \frac{dP}{d\Omega} = \frac{c}{8\pi}\mathrm{Re}[\hat{\mathbf{n}} \cdot (\vec{\mathbf{E}} \times \vec{\mathbf{B}}^*)].$$

(b) Integrate your result in (a) to find the total scattered power (in all directions) and show that the total scattered power is

(i) proportional to the fourth power of the frequency if the particle is massless (*i.e.* for $m = 0, \eta \neq 0$); and

(ii) frequency independent if the particle is unrestrained (*i.e.* for $m \neq 0, \eta = 0$).

The Debye theory treats a crystal as a continuum, and hence the phonon dispersion relation is $\omega = ck$, where c is the speed of sound in the body. In a ferromagnetic solid at low temperatures, there also exist quantized waves of magnetization (magnons) for which the dispersion relation is of the form $\omega = bk^2$, where b is a constant. In both these cases a large-k (small wavelength) cutoff must be imposed. Take the cutoffs in these two cases as k_D and k_M, respectively.

Obtain expressions valid near $T=0$ for the phonon and magnon contributions to the specific heat.

Note: Your answers may include a *dimensionless* multiplicative constant in the form of an integral that you *need not* to evaluate. The main point is to obtain the temperature dependence of the two contributions to the specific heat at low temperatures.

Consider an Ising spin system consisting of three spins ($s = ½$) on a line with nearest-neighbor interactions. Each spin has a magnetic moment $\vec{m} = 2\mu\vec{s}$ pointing in the same direction as the spin. The system is in an external magnetic field H in the z direction and is in equilibrium at a temperature T. The Hamiltonian for the system is

$$\mathscr{H} = J\,S_z(1)\,S_z(2) + J\,S_z(2)\,S_z(3) - 2\mu H\,[S_z(1) + S_z(2) + S_z(3)]\ ,$$

where J and μ are positive constants.

(a) List each of the possible microstates of the system and the corresponding energy. Sketch the energy-level diagram for the case where $2\mu H > J/2 > \mu H$ indicating any degeneracy.

(b) For each of the following conditions write down the limiting values for the internal energy $U(T,H)$, the entropy $S(T,H)$ and the magnetization $M(T,H)$:
- (i) $T = 0$ and $H = 0$
- (ii) $T = 0$ and $0 < H \ll J/\mu$
- (iii) $T = 0$ and $J/\mu \ll H$
- (iv) $J \ll kT$ and $H = 0$

(c) On the basis of simple physical considerations, obtain the limiting temperature dependence at high and at low temperatures of the specific heat at constant field $C_H(T,H)$ when $H = 0$. Sketch C_H vs. T for $H = 0$.

(d) Obtain the partition function for the system.

(e) Find the magnetization $M(T,H)$ and show that, in the limit $kT \gg \mu H$ or $kT \gg J$, it reduces to $M \approx 3\mu^2 H/kT$.

Consider a particle of mass m and electric charge q, which can only move in one dimension, subjected to a harmonic force and a homogeneous electrostatic field E_0. The Hamiltonian for the system is given by $H = H_{osc} + H_{el} = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2x^2 - q\,E_0 x$.

(a) Determine the energy eigenvalues and eigenkets for vanishing field (i.e. E_0=0) using the ladder operators a and a^+ with $a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right)$.

(b) Find the energy eigenvalues and eigenkets for the system when $E_0 \neq 0$ by considering the translation $x \to x + l$ in the Hilbert space where $l = \frac{qE_0}{m\omega^2}$. *Hint*: A translation in the coordinate space is generated by the momentum operator ($\exp(-\frac{i}{\hbar}pl)$).

(c) If the particle is initially in the ground state of the unperturbed oscillator H_{osc}, what is the probability of finding it in the ground state of the full Hamiltonian H?

MS Comprehensive Examination
Fall 2008
Quantum Mechanics 600-2

Consider a neutron scattering off a hydrogen molecule (H_2). All three particles, the neutron and both protons, are fermions (spin $s = ½$). The central-force $n - p$ interaction can be written as

$$V = \tfrac{1}{2}(3V_t + V_s) + \tfrac{1}{\hbar^2}(V_t - V_s)(\vec{S}_n \cdot (\vec{S}_{p1} + \vec{S}_{p2})),$$

where $\vec{S}_i$ denote the spin operators of the three particles and $V_s(r)$ and $V_t(r)$ the scalar and tensor components of the potential.

(a) Find the eigenvalues of the total spin and construct the eigenkets of the total spin operators $\vec{\mathbf{S}}^2$ and S_z.

(b) Use this eigenket system to determine the contributions of V_t and V_s to the $n - p$ interaction potential V for para H_2 (anti-symmetric spin orientation) and ortho H_2 (symmetric spin orientations).