CUA

THE CATHOLIC UNIVERSITY OF AMERICA

Department of Physics
200 Hannon Hall
Washington, D.C. 20064
202-319-5315
Fax 202-319-4448

Preliminary Examination

Fall 2005

Thursday, October 27, and Friday, March 28, 2005

Room 135 - Hannan Hall

- **YOU <u>MUST</u> DO TWO QUESTIONS IN EACH OF THE AREAS:**

<u>Thursday, October 27, 2005</u>	Mechanics
	Electricity & Magnetism
<u>Friday, October 28, 2005</u>	Thermodynamics
	Modern Physics/Quantum Mechanics

- **DO EACH PROBLEM IN A SEPARATE BLUE BOOK**

- **PUT YOUR NAME ON EACH BLUE BOOK**

- **LABEL EACH BOOK WITH CORRECT PROBLEM NUMBERS**

  FOR EXAMPLE: Mechanics #1

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MS Comprehensive Examination
Physics Department

Fall 2005

Thursday, October 27 and Friday, October 28, 2005

Room 135 - Hannan Hall

GENERAL INSTRUCTIONS:

This examination is divided into four sections as follows:

Thursday, October 27, 2005

9:00 a.m. - 12:00 Noon	Classical Mechanics - 2 questions
1:00 P.M. - 5:00 P.M.	E & M - 2 questions

Friday, October 28, 2005

9:00 a.m. - 12:00 Noon	Thermodynamics/Stat. Mech. - 2 questions
1:00 P.M. - 5:00 P.M.	Modern Physics/Quantum Mech. - 2 questions

DO EACH PROBLEM IN A SEPARATE BLUE BOOK

PUT YOUR NAME ON EACH BOOK

LABEL EACH BOOK WITH CORRECT PROBLEM NUMBERS
FOR EXAMPLE: Mechanics 600-1 or Electromagnetism 500-2

YOU MUST DO TWO (AND ONLY TWO) OF THE QUESTIONS IN EACH AREA.

OF THE TOTAL OF EIGHT QUESTIONS THAT ARE DONE, A MINIMUM OF TWO MUST BE FROM THE 600-LEVEL QUESTIONS.

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Department of Physics
200 Hannon Hall
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Ph.D. Comprehensive Examination

Physics Department

Fall 2005

Thursday, October 27, and Friday, October 28, 2005

Room 135 - Hannan Hall

GENERAL INSTRUCTIONS:

This examination is divided into four sections as follows:

<u>Thursday, October 27, 2005</u>

9:00 a.m. - 12:00 Noon	Classical Mechanics - 2 questions
1:00 P.M. - 5:00 P.M.	E & M - 3 questions

<u>Friday, October 28, 2005</u>

9:00 a.m. - 12:00 Noon	Stat Mech. - 2 questions
1:00 P.M. - 5:00 P.M.	Quantum Mech. - 3 questions

<u>DO EACH PROBLEM IN A SEPARATE BLUE BOOK</u>

PUT YOUR NAME ON EACH BOOK

LABEL EACH BOOK WITH CORRECT PROBLEM NUMBERS
FOR EXAMPLE: Mechanics 600-1

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RULES FOR THE PHYSICS COMPREHENSIVE EXAMINATION

FALL 2005

In order to assure an equitable and fair Comprehensive Examination for all students the following rules will apply to the examination administered by the Department of Physics.

1. The examination will be closed book. Students will not be permitted to bring to the examination room materials such as notes, reference books, calculators, or other aids of any form. The Physics Department will supply copies of the *CRC Mathematical Handbook,* Schaum's *Mathematical Handbook, Table of Functions* by Jahnke and Emde, and the NBS *Handbook of Mathematical Functions*, for the use of those taking the exam should they feel the need for these references during the examination.

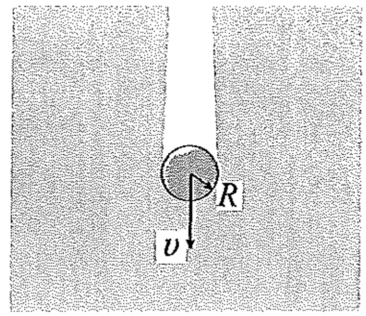
   The Physics Department will supply calculators for use during the examination.

   Students who want to use their own mathematical tables and/or handbooks must get permission from the Chairman prior to the beginning of the comprehensive examination. These tables and/or handbooks will remain under the control of the department until after the examination is completed, but will be available to their owners during the exam.

2. If needed, students will be allowed a short break, not to exceed ten minutes, during the examination period. While absent from the examination room students may not consult material or other sources of information concerning any matter pertaining to the examination. Students who leave the examination room will turn in all of their examination materials to the proctor before leaving the room. Examination papers will be returned to the students when they reenter the examination room. Students who fail to return within ten minutes will be considered to have completed that portion of the examination.

   Only one student will be permitted to leave the examination room at a time.

A small spherical hail pellet is formed in the fine water mist in a cloud. It falls toward the earth sweeping up the mist in its path, thereby increasing its mass. Make the following simplifying assumptions.



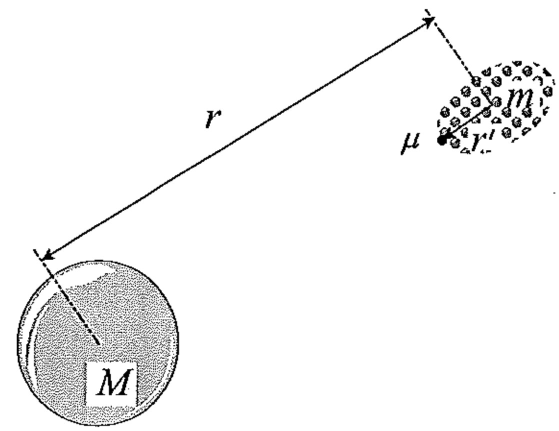
- All the mist *along volume of circular cross section* swept out by the pellet is retained on the pellet as ice.
- The pellet remains spherical as it falls.
- The pellet experiences no viscous drag during its descent.
- The density of the ice pellet ρ_P is much larger than the density of the mist ρ_M; thus, the buoyant force on the pellet can be ignored.
- The velocity of the pellet is given by

  $$v(t) = at$$

  where a is its constant acceleration ($a < g$) and t is time.

(a) Show that the radius of the pellet R is of the form $R = bt^2 + c$, where b and c are constants.
(b) Calculate the value of the acceleration a.

When a swarm of gravitationally bound particles approaches too close to a massive object, M, the swarm tends to be torn apart. Consider a gravitationally bound swarm, having a total mass m, falling straight towards mass M. The distance between the centers of mass of the swarm and the large mass, M, is r.



(a) For a particle of mass μ at the surface of the swarm closest to M, a distance r' from the center of mass of the swarm ($r' \ll r$), show that the condition for it to be pulled steadily away from the swarm is:

$$\frac{2M}{r^3} > \frac{m}{r'^3}.$$

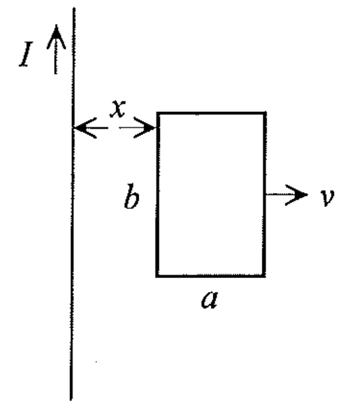
Keep only the terms to first order in r'. This phenomenon is known as *tidal disruption.*

(b) Now, consider the case when the swarm is moving in a perfectly circular orbit around mass M. Assume the swarm is not rotating about its center. Show that the condition for disruption is now:

$$\frac{3M}{r^3} > \frac{m}{r'^3}.$$

It is helpful in part *(b)* to recognize that the center of mass of the swarm has a different centripetal acceleration from the particle of mass μ at the surface of the swarm.

A long straight wire carries a constant current I. A rectangular loop of wire (of width a and height b) lies in the same plane as the wire, and is being pulled away from the wire at constant speed v.



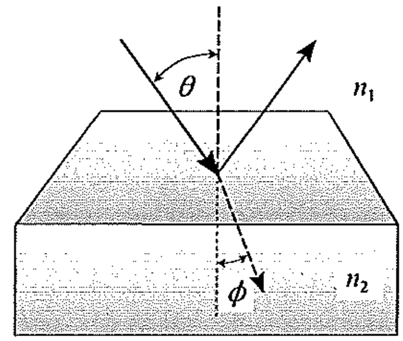
a) Calculate the **magnetic flux** through the wire loop due to the magnetic field of the long wire when the near side of the loop is a distance x from the wire.

b) From the result of (a), calculate the EMF induced in the loop when it is a distance x from the wire. (Note that $dx/dt = v$.)

c) Let R be the resistance of the wire loop. Calculate the **current** induced in the loop, and tell in which direction it flows (clockwise or counterclockwise.)

d) Find the net **force** required to keep the loop moving at constant speed v when it is at distance x from the long wire.

Consider a plane electromagnetic wave incident at angle θ on a plane boundary separating two non-conducting media with indices of refraction n_1 and n_2, as shown. The space and time dependences of the incident, reflected, and transmitted waves have the forms: $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, $e^{i(\vec{k}'\cdot\vec{r}-\omega t)}$, and $e^{i(\vec{k}''\cdot\vec{r}-\omega t)}$, respectively.



(a) Write the expressions for the electric and magnetic fields of the incident wave.

(b) For the case in which the magnetic vector of the incident wave is parallel to the boundary (polarization in the plane of incidence) show that the ratio of the reflected to the incident amplitude of the wave, r_p, is as follows:

$$r_p = \frac{-n\cos\theta + \cos\phi}{n\cos\theta + \cos\phi}, \text{ where } n = n_2/n_1.$$

HINT: Apply the boundary conditions.

(c) Using Snell's law, show that the expression for r_p can be rewritten as:

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}.$$

(d) Consider the case for which $n < 1$. In this case show that, when the angle of incidence

$$\theta > \sin^{-1} n,$$

the square of the amplitude of reflection coefficient r_p becomes unity. Describe what this means physically.

Consider a rigid cube of volume $V=L^3$ filled with n moles of an ideal gas. Each molecule has a mass m. Assume that a molecular collision with any wall of the enclosure is completely elastic.

(a) Show that the average rate $(\Delta p_i/\Delta t = dp/dt)$ at which each molecule delivers momentum to any wall of the enclosure is:

$$dp_i/dt = m \langle (\upsilon_i^2) \rangle / L.$$

Here υ_i is the x-, y-, or z-component of the velocity of the molecule. The brackets $\langle ... \rangle$ indicate a thermal average.

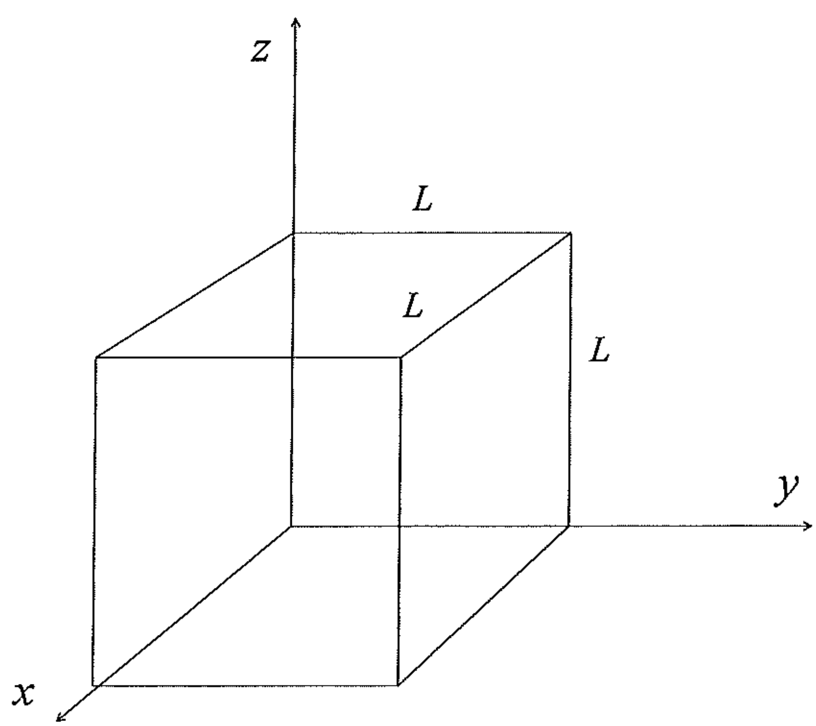
(b) Show that the pressure, P, is given by

$$P = (nM/3V)\,(\upsilon_{rms})^2,$$

where $M = mN_a$ and N_a is Avogadro's number.

Remember that $\upsilon_{rms} = \langle \upsilon^2 \rangle^{1/2}$, where $\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2$.

(c) Finally, show that $\upsilon_{rms} = (3\,N_a k\,T/M)^{1/2}$, where k is Boltzmann's constant.



Consider a classical ideal monatomic gas in thermal equilibrium at temperature T.

(a) Write the expression for the velocity distribution function of the atoms.
(b) Derive the expression for the *most probable speed* of the atoms.
(c) If the gas is atomic hydrogen at a temperature $T = 6400$ K, estimate the value of the most probable speed. The mass of a hydrogen atom is $m = 1.67 \times 10^{-27}$ kg and Boltzmann's constant is $k = 1.38 \times 10^{-23}$ J·K^{-1}.
(d) Calculate the average kinetic energy per atom of this gas. What is the rms speed of an atom in the gas?
(e) Now consider a stream of this ideal gas moving at a constant velocity $\vec{v}_0$. Write the expression for the velocity distribution in the laboratory frame.
(f) How does the streaming affect your answers to parts *(b)*, *(c)*, and *(d)* in the rest frame of the gas?

For a spin-1/2 particle (such as an electron), the three components of spin angular momentum can be written as $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$, where the components of $\vec{\sigma}$ are the Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Write all the commutators of the components S_x, S_y and S_z (i.e. $[S_x, S_y]$, $[S_y, S_z]$, ... What does this tell you about whether you can simultaneously determine the components S_y and S_z?

(b) An arbitrary spin state may be written as $\chi = a\chi_+ + b\chi_- = \begin{pmatrix} a \\ b \end{pmatrix}$, where χ_+ and χ_- are the normalized eigenvectors of S_z, and a and b are complex numbers. Find the expectation values of S_x, S_y and S_z for such a state.

(This part does not depend on parts *(a)* or *(b)*.)

(c) Consider a system of 3 electrons. The total spin angular momentum of the system is

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 .$$

What are the possible values that you might measure for

(1) $S^2 = \vec{S}\cdot\vec{S}$

(2) S_z (the z-component of $\vec{S}$)

(Hint: First combine the spins of two electrons, then add the third to the sum.)

Ultraviolet radiation uniformly and normally illuminates an aluminum surface of area 1.0 cm^2. The work function for aluminum is 4.2 eV.

(a) What is the maximum wavelength radiation that will cause electrons to be emitted from the surface?

(b) If incident radiation of wavelength 150 nm illuminates the surface, compute the kinetic energy of the fastest emitted photoelectrons?

(c) Suppose that the quantum efficiency is 2% at this wavelength (that is, 1 out of every 50 photons causes an electron to be ejected). If the irradiance of the normally incident radiation is 2.0 Watts/m^2, how many electrons are emitted per second?

(d) Suppose one treats the incident radiation as a classical electromagnetic wave rather than as a stream of photons. At a wavelength of 150 nm the skin depth for aluminum is less than 10^{-10} m, so that only electrons very near the surface can respond to the radiation. Assume that the surface density of such electrons is 10^{15} electrons/cm^2. For an incident beam with an irradiance of 2.0 Watts/ m^2, what would be the approximate time delay before electrons would be emitted? If the wavelength were doubled, how would this time delay change? Assume that all of the incident radiation is absorbed by the surface electrons.

Numerical data: $e = 1.60 \times 10^{-19}$ C
$h = 4.14 \times 10^{-15}$ eV·s.
$c = 3.00 \times 10^{8}$ m/s

A particle of mass m moves on the inside wall of a frictionless vessel, which is axially symmetric about the z-axis with a surface defined by

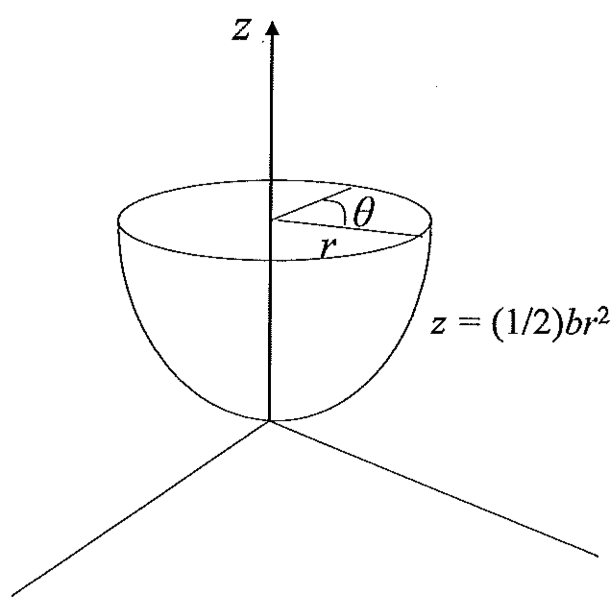
$$z = \frac{1}{2} b r^2,$$

where b is a constant and z is the vertical direction (See figure below.) The particle is moving in a circular orbit around the inside of the vessel at fixed height $z = z_o$.

(a) Obtain the total energy (kinetic + potential) and angular momentum of the particle in terms of z_0, b, and the gravitational acceleration g.

(b) Obtain Lagrange's equations of motion using cylindrical coordinates, r and θ, as generalized coordinates.

(c) The particle in the horizontal circular orbit is perturbed slightly. Obtain the frequency of oscillation about the unperturbed orbit for very small oscillations. Again, give your answer in terms of z_o, b, and g.



Consider a collection of particles whose position vectors $\vec{r}_i$ and momenta $\vec{p}_i$ are both bounded (*i.e.*, remain finite for all time).

(a) Derive the ***virial theorem*** $\langle K \rangle = -\frac{1}{2} \langle \sum_i \vec{F}_i \cdot \vec{r}_i \rangle$ where $\langle K \rangle$ is the average kinetic energy of the system and $\vec{F}_i$ is the force on the i^{th} particle.

Begin by defining a quantity $S \equiv \sum_i \vec{p}_i \cdot \vec{r}_i$ and then take its time derivative dS/dt. Then determine its average value, $\langle dS/dt \rangle$ over a time interval τ. Note that if the system's motion is periodic and τ is some integer multiple of the period, $\langle dS/dt \rangle$ will vanish. However, even if the system does not display any periodicity, one can make $\langle dS/dt \rangle$ as small as desired by making τ sufficiently long, that is by letting $\tau \to \infty$. By taking this limit, the virial theorem follows.

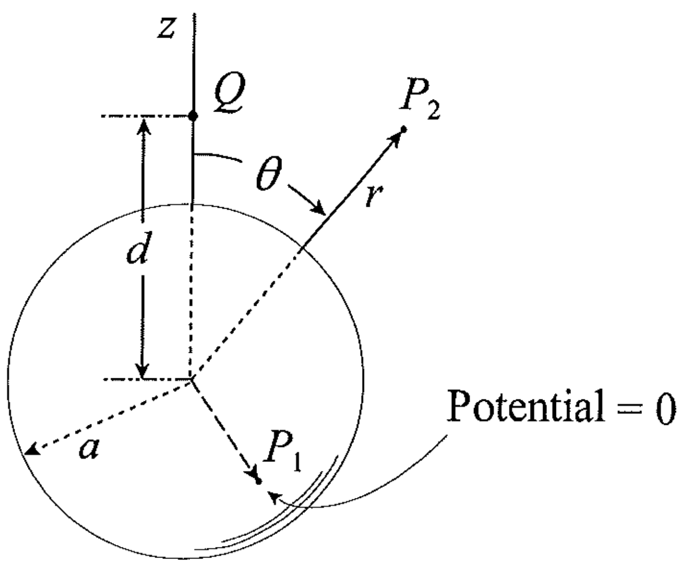
(b) Consider an ideal gas containing N atoms in a container of volume V, pressure P, and absolute temperature T. The only force involved is the force of constraint by the walls of the container. Use the virial theorem to derive the equation of state for a perfect gas:

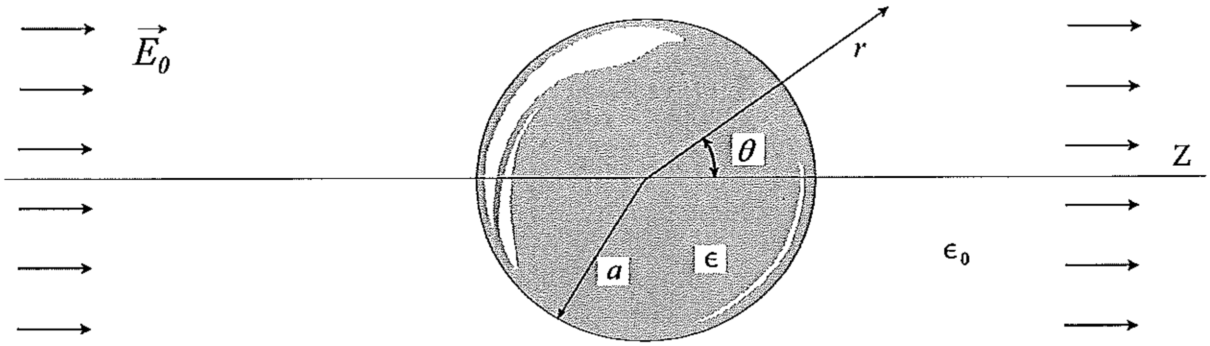
$$PV = NkT,$$

where k is Boltzmann's constant. Note, according to the equipartition theorem, the average kinetic energy of each atom in the ideal gas is $\frac{3}{2}kT$.

Consider a point charge, Q, at a distance d from the center of a grounded conducting sphere of radius a (see the figure below). Using the method of images:

(a) Find the magnitude and position of the image charge.
(b) Show explicitly that the resulting charge arrangement will make the potential zero at a general point P_1 on the surface of the sphere (as shown).
(c) Determine the potential at an arbitrary point P_2 outside the sphere at a position (r, θ).
(d) Find the electric field at the point P_2.
(e) Determine the induced charge density, σ, on the surface of the sphere.
(f) Now, consider the case when the sphere is at a potential V other than zero. What are the position(s) and magnitude(s) of the image charges?





A dielectric sphere of radius a and permittivity ϵ is placed in an initially uniform electric field $\vec{E}_0$ in a vacuum. There are no free charges.

(a) Find the electric field $\vec{E}(r, \theta)$ both inside and outside the sphere.
(b) What is the polarization $\vec{P}$ inside the sphere?
(c) What is the surface charge density due to the polarization?
(d) How would the answers to parts *(a)*, *(b)*, and *(c)* be modified if the medium outside the sphere were a dielectric with permittivity $\epsilon' > \epsilon$?

A harmonic plane wave polarized in the x-direction propagates in the z-direction in charge-free empty space. The wave has a gaussian profile

$$E_0(x,y) = E_0 \, e^{-\alpha^2(x^2+y^2)}$$

and an angular frequency ω. Assume that the amplitude modulation is slowly varying (the wave is many wavelengths broad), that is, $k >> \alpha$.

(a) Show that the electric field is given approximately by

$$\vec{E}(x,y,z,t) \approx E_0 \left[\hat{e}_1 - \hat{e}_3 \frac{2i\alpha^2 x}{k} \right] e^{-\alpha^2(x^2+y^2)} e^{ikz - i\omega t},$$

where $\hat{e}_1$ and $\hat{e}_3$ are unit vectors in the x- and z-directions, respectively.

(b) Obtain a corresponding expression for the magnetic field.

(c) How can you reconcile the existence of a z-component of the electric field with the assertion of the gospel according to Maxwell that electromagnetic waves are transverse?

HINT: Make sure that Maxwell's equations are satisfied to first order in α / k.

A cylinder of radius R and length L rotates about its axis with a constant angular velocity ω. Ignore the effects of gravity and treat the system classically assuming that thermal equilibrium is established at temperature T.

(a) Obtain an expression for the density distribution of a classical ideal monatomic gas (N atoms in equilibrium at temperature T) enclosed in the cylinder; that is, calculate the density ρ as a function of r, where r is the radial coordinate measured from the cylinder axis.

(b) What can you infer qualitatively from this about the distribution of gas in the cylinder?

HINT: The Hamiltonian that describes the system is $H = H_0 - \omega \mathcal{L}$ where H_0 is the Hamiltonian that one would obtain if the system were not rotating and

$$\mathcal{L} = \sum_{i=1}^{N} (x_i p_{yi} - y_i p_{xi})$$

denotes the system's angular momentum. In this expression x_i and p_{yi} are the x-component of the position coordinate and the y-component of momentum for the i^{th} atom, respectively. Similarly, y_i and p_{xi} are the y-component of the position coordinate and the x-component of momentum for that atom. Apply the canonical distribution for H.

Show that the internal energy of an ideal Bose gas in a volume V at a temperature T is given by

$$E = \frac{3}{2} VkT \left(\frac{2\pi\, m k T}{\hbar^2} \right)^{3/2} \sum_{n=1}^{\infty} \frac{e^{n\mu/kT}}{n^{5/2}}$$

when the degeneracy is weak, that is, when the magnitude of the chemical potential $|\mu| \gg kT$ (recall that if the lowest value of the single particle energy is taken to be zero, then $\mu \leq 0$). The other symbols have their usual meanings: m is the mass of one of the gas molecules, k is Boltzmann's constant, and $\hbar$ is Planck's constant divided by 2π.

To solve this problem, begin with the expression for the energy of an ideal Bose gas

$$E = \sum_s \frac{\epsilon_s}{e^{(\epsilon_s - \mu)/kT} - 1},$$

and convert the sum over states s to an integral over energy as

$$\sum_s \rightarrow \int d\epsilon\, D(\epsilon),$$

where $D(\epsilon)$ is the density of states. Display $D(\epsilon)$ explicitly, and then use it to obtain E as required.

HINT 1: The expansion $\dfrac{1}{1-z} = \sum_{n=0}^{\infty} z^n$ for $|z| < 1$ may prove to be useful.

HINT 2: $$\int_0^{\infty} dx\, x^{(m - ½)} e^{-x} = \Gamma(m + ½) = \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{2^m} \sqrt{\pi}.$$

Consider the harmonic oscillator quantum state with the wave function

$$\Psi(x,t) = \sqrt{\frac{1}{3}}\left[\psi_0(x)\,e^{-iE_0t/\hbar} + \psi_1(x)\,e^{-iE_1t/\hbar} + \psi_2(x)\,e^{-iE_2t/\hbar}\right]$$

where $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are taken as real, normalized eigenfunctions of the harmonic oscillator with energy E_0, E_1 and E_2 respectively (the first three stationary states).

(a) Find the expectation value of the energy in terms of the classical oscillator frequency $\omega = \sqrt{k/m}$.

(b) Find the uncertainty in the energy.

(c) Find the probability density for finding the particle at position x as a function of position x and time t. Leave your answer in terms of the functions $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$, but calculate and simplify the time dependence. Is the time variation periodic? If so, what is the period?

(a) Consider a system described by a Hamiltonian $\mathcal{H}(p_1, p_2 \ldots, q_1, q_2 \ldots) = \mathcal{H}\{p,q\}$. Let $\psi\{q\}$ be *any* normalized function of the system coordinates $\{q\}$, that is

$$\int dq_1 \int dq_2 \; \ldots \; \psi\{q\}^* \; \psi\{q\} \; = \; 1.$$

Prove that

$$E' \equiv \langle \psi | \mathcal{H} | \psi \rangle \geq E_G$$

where E_G is the ground state energy of the system. This is the basis for the so-called ***variational method*** of estimating the ground state energy. One chooses a *trial wave function* that depends on some parameter λ in addition to the coordinates $\{q\}$; an upper bound can then be found for E_G by varying λ to minimize E'.

(b) As an example, apply the variational method to calculating the ground state energy of the anharmonic oscillator described by the Hamiltonian

$$\mathcal{H} = \frac{p^2}{2m} + \kappa x^4,$$

where p denotes the momentum operator, m is the mass, κ is a constant, and x is the departure of the oscillator from its equilibrium position. Use a trial function of the form of the harmonic oscillator ground state eigenfunction (suitably parameterized)

$$\psi(x;\lambda) = \lambda^{1/2} \pi^{-1/4} e^{-\lambda^2 x^2/2},$$

which is already normalized ($\langle \psi | \psi \rangle = 1$).

HINT: In carrying out the calculation, you may find the following to be of help:

$$\int_{-\infty}^{\infty} dx \, x^{2n} e^{-\lambda^2 x^2} = \frac{\sqrt{\pi}}{\lambda} \qquad (n = 0)$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n \lambda^{2n+1}} \qquad (n = 1, 2, \ldots)$$

Consider the spin angular momentum matrices for a particle of **spin 1**. Since there are three possible projections of $\vec{S}$ in the z-direction, the eigenfunctions of S^2 and S_z form a vector space of dimension 3.

(a) Write the matrices representing the observables S^2 and S_z in a representation in which the basis vectors are the eigenfunctions of S_z. Let the top row of each matrix correspond to the largest positive eigenvalue of S_z.

(b) In this basis, the spin components S_x and S_y have the matrix representations

$$S_x = \begin{pmatrix} 0 & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & 0 & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & 0 \end{pmatrix}, \qquad S_y = \begin{pmatrix} 0 & -i\frac{\hbar}{\sqrt{2}} & 0 \\ i\frac{\hbar}{\sqrt{2}} & 0 & -i\frac{\hbar}{\sqrt{2}} \\ 0 & i\frac{\hbar}{\sqrt{2}} & 0 \end{pmatrix}.$$

By diagonalization (or another method of your choice), find the **eigenvalues and eigenvectors** of S_x (in the basis of eigenvectors of S_z), and verify that they satisfy the eigenvalue equation. Show explicitly that S_x can be diagonalized by a similarity transformation.

(c) In the original basis (in which the basis vectors are the eigenfunctions of S_z), write the matrices corresponding to the raising and lowering operators S_+ and S_-. Show explicitly that they have the desired effect when applied to the eigenvectors of S_z, and also that (when properly normalized) they agree with the usual relationships $S_\pm = S_x \pm iS_y$.

(d) (This part is not dependent on parts *(a)*, *(b)* or *(c)*.)
If a spin-1 particle has orbital angular momentum quantum number $l = 2$, what are the possible observable values of J^2 and J_z (where $\vec{J} = \vec{L} + \vec{S}$)?