CUA

THE CATHOLIC UNIVERSITY OF AMERICA

Department of Physics
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THE CATHOLIC UNIVERSITY OF AMERICA

Preliminary Examination--Physics Dept.

Thursday, October 24, and Friday, October 25, 2002

Room 133 - Hannan Hall

- **YOU <u>MUST</u> DO TWO QUESTIONS IN EACH OF THE AREAS:**

<u>Thursday, October 24, 2002</u>

Mechanics

Electricity & Magnetism

<u>Friday, October 25, 2002</u>

Thermodynamics

Modern Physics/Quantum Mechanics

- **DO EACH PROBLEM IN A SEPARATE BLUE BOOK**

- **PUT YOUR NAME ON EACH BLUE BOOK**

- **LABEL EACH BOOK WITH CORRECT PROBLEM NUMBERS**

  FOR EXAMPLE: Mechanics #1

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RULES FOR THE PHYSICS PRELIMINARY EXAMINATION

Fall 2002

In order to assure an equitable and fair Preliminary Examination for all students the following rules will apply to the examination administered by the Department of Physics.

1. The examination will be closed book. Students will not be permitted to bring to the examination room materials such as notes, reference books, calculators, or other aids of any form. The Physics Department will supply copies of the *CRC Mathematical Handbook,* Schaum's *Mathematical Handbook, Table of Functions* by Jahnke and Emde, and the NBS *Handbook of Mathematical* Functions, for the examinees' use should they feel the need for these references during the examination.

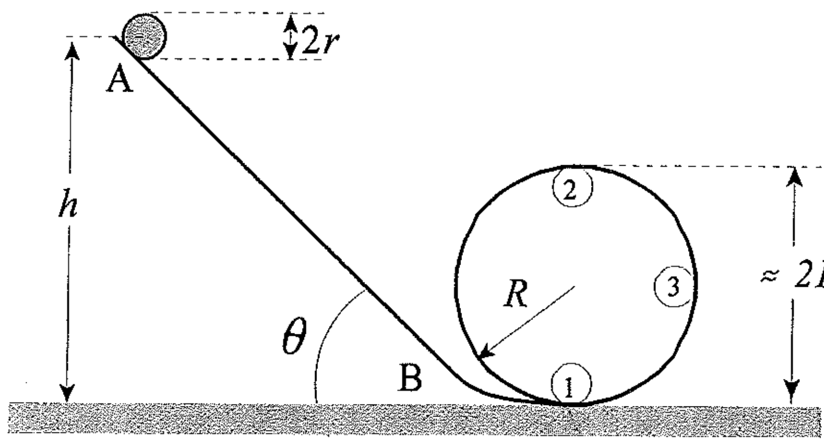
   Students who want to use their own mathematical tables and/or handbooks must get permission from the Chairman at least two days before the beginning of the comprehensive. These tables and/or handbooks will remain under the control of the department until after the comp exam but will be available to their owners during the exam.

2. If needed, students will be allowed a short break, not to exceed 5 minutes, to use the restroom facilities during the examination period. While absent from the examination room students may not consult material or other sources of information concerning any matter pertaining to the examination. Students who leave the examination room will turn in all of their examination materials to the proctor before leaving the room. Examination papers will be returned to the students when they reenter the examination room. Students who fail to return within 5 minutes will be considered to have completed that portion of the examination.

<u>Only one student will be permitted to leave the examination room at a time.</u>

Preliminary Examination
Fall 2002
Mechanics 500-1

A uniform solid cylinder of radius r and mass m starts at rest at point A and rolls without slipping down the track, around the circular loop (radius = $R \gg r$), and continues rolling to point C and beyond. From A to B the track is straight and is inclined at an angle θ with respect to the horizontal.



(a) Rolling without slipping requires static friction. What is the minimum coefficient of static friction required for the cylinder to roll without slipping along the straight portion of the track?

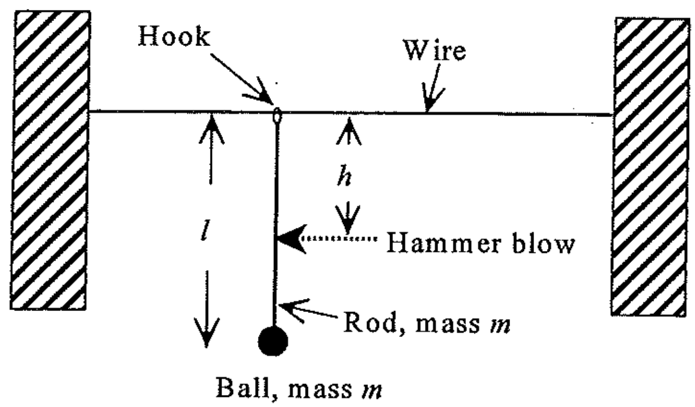
HINT: The rotational inertia (moment of inertia) for a solid cylinder of mass m, radius r and uniform density about its symmetry axis is $I = mr^2/2$

(b) What is the minimum starting height h that is needed if the cylinder is to roll around the loop without falling off? You may assume that the cylinder's height at the top of the loop is approximately $2R$ since $R \gg r$.

(c) Assume that h is greater than the value calculated in part *(b)*.
What are the normal forces that the track exerts on the cylinder at
(1) the bottom of the loop,
(2) the top of the loop, and
(3) halfway up the loop
(the three positions indicated)?

Preliminary Examination
Fall 2002
Mechanics 500-2

A uniform thin rod of mass m and length l hangs vertically, at rest, from a hook which can slide freely along a horizontal frictionless wire, as shown. Attached to the end of the rod is a small ball, also of mass m.



a) Find the location of the center of mass for the rod-plus-ball system, taking the hook as the origin of the coordinate system.

b) Find the moment of inertia about the hook for the rod-plus-ball system.

c) Use the result of part (b) and the parallel-axis theorem to find the moment of inertia of the rod-plus-ball system about its center of mass.

The rod is struck by a horizontal hammer blow (in the plane of the rod and wire, as shown) at a distance h from the hook, imparting an impulse ΔP to the rod-plus-ball system.

d) Find the **center-of-mass velocity** and the **angular velocity** of the rod-plus-ball system at the instant just after the hammer blow.

e) Find the value of h such that the hook does not move along the wire at the instant of the hammer blow.

(Note: Part (f) does not depend on the result of parts (d) and (e).)

f) Write the Lagrangian for the motion of the rod-plus-ball system.
Suggested coordinates:

X_{CM} = horizontal position of the center of mass
θ = angle of the rod relative to the vertical

A plane electromagnetic wave propagates in an uncharged, linear, homogeneous, isotropic medium characterized by a dielectric permittivity ε, magnetic permeability μ and electrical conductivity σ. Assume that all fields vary harmonically in time, that is, that they are of the form

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\,e^{-i\omega t}.$$

(a) Beginning with Maxwell's equations, derive a Helmholz equation, that is an equation of the form

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

for $\mathbf{E}(\mathbf{r})$.

(b) Show that k is complex, that is $k = \kappa + i\alpha$ when the electrical conductivity $\sigma \neq 0$.

(c) Show that

$$\mathbf{E}(z,t) = \mathbf{E}_0\, e^{i(kz - \omega t)}$$

is a solution of the differential equation above.

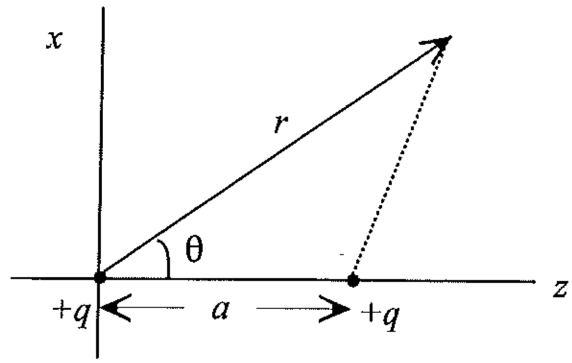
(d) Use Maxwell's equations to prove that this is a transverse wave (first define what is meant by a transverse wave).

(e) What is the phase angle φ between **E** and **H** when $\kappa \gg \alpha$?
HINT: When $\alpha = 0$, **E** and **H** are in phase ($\varphi = 0$). In answering this question, you are to retain only the lowest order term in α / κ.

(f) What is the intensity I of the wave in this case (*i.e.*, when $\kappa \gg \alpha$)?
HINT: Take $I = \langle |\mathbf{S}| \rangle$ where **S** is the Poynting vector and $\langle ... \rangle$ denotes a time average. Again, retain only the lowest order term in α / κ.

Preliminary Examination
Fall 2002
Electricity & Magnetism 500-2

Two equal positive point charges $+q$ are located on the z axis at $z = 0$ and $z = a$.



(a) Write an exact expression for the electric potential $V(r,\theta)$ at any point in space. (It may be helpful to start with coordinates x, y, z and convert to r,θ.)

(b) Write exact expressions for all components of the electric field $\boldsymbol{E}$ at the arbitrary point in the x-z plane (x, 0, z).

(c) Starting with the exact potential of (a), make an expansion of the potential in powers of r which is valid at large distances ($r \gg a$), keeping terms of order r^{-1} and r^{-2} only.

(d) What is the electric dipole moment of this charge distribution? Explain your reasoning. (There are several possible valid methods.)

A mass m of ice at temperature T_1 is added to an equal mass of water at T_2 and the mixture is allowed to reach equilibrium where all the ice is melted and the final temperature is T_3.

(a) Show that the final equilibrium temperature is

$$T_3 = \frac{1}{2\,c_p^w}\left[(c_p^w - c_p^I)T_{ice} + c_p^w T_2 + c_p^I T_1 - L\right],$$

where c_p^w and c_p^I are the heat capacities of water and ice, respectively (assumed constant), L is the latent heat of fusion of ice at its melting point and T_{ice} is the melting temperature of ice.

(b) Show that the change in the entropy of the universe is:

$$\Delta S = m\left[c_p^I \ln\frac{T_{ice}}{T_1} + \frac{L}{T_{ice}} + c_p^w \ln\left(\frac{T_3^2}{T_{ice}T_2}\right)\right].$$

A building is maintained at an equilibrium temperature T by means of a heat pump which uses a river at temperature T_0 as a source of heat. Assume that the heat pump is a ideal Carnot engine and consumes energy at a constant rate (power P = *constant)*, and the building loses heat Q to its surroundings at a rate $\alpha\,(T - T_0)$, where α is a constant, that is

$$\frac{dQ}{dt} = -\alpha(T - T_0).$$

(a) Define and calculate the thermodynamic efficiency of the heat pump operating between T and T_0.

(b) Show that the equilibrium temperature of the building, T, is given by:

$$T = T_0 + \frac{P}{2\alpha}\left[1 + \left(1 + \frac{4\alpha T_0}{P}\right)^{1/2}\right].$$

The eigenfunctions for a simple harmonic oscillator of mass m, angular frequency ω and potential energy

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

are of the form

$$\psi_n(x) = N_n e^{-\xi^2/2} H_n(\xi) \quad (n = 0, 1, 2, 3 \ldots)$$

where $\xi = \sqrt{\alpha}\,x$ with $\alpha = m\omega/\hbar$ and $N_n = \left[\frac{\sqrt{\alpha/\pi}}{2^n\, n!}\right]^{1/2}$. The H_n are the Hermite polynomials, the first few of which are

$$H_0(\xi) = 1$$
$$H_1(\xi) = 2\xi$$
$$H_2(\xi) = 4\xi^2 - 2$$
$$H_3(\xi) = 8\xi^3 - 12\xi.$$

(a) Calculate the energy eigenvalues for the ground and first excited states by plugging the appropriate eigenfunctions into the time-independent Schroedinger equation.

(b) Use first order time-independent perturbation theory to calculate the ground state and first excited state energies for a one-dimensional system consisting of a single particle of mass m moving under the influence of a potential energy given by

$$V(x) = \frac{1}{2}m\omega^2 x^2 + \lambda x^4,$$

with λ a constant.

Preliminary Examination
Fall 2002
Quantum Mechanics 500-2

a) The energy of an electron in an atom is shifted by the so-called spin-orbit interaction, which results in a term of the form $\Delta E = A\langle\vec{L}\cdot\vec{S}\rangle$, where L is the orbital angular momentum, S is the spin angular momentum, A is a constant, and the brackets $\langle\ \rangle$ denote the expectation value in a particular quantum state. Let the total angular momentum of the electron be $\vec{J} = \vec{L} + S$. The magnitudes of the angular momenta $\vec{L}$, $\vec{S}$ and J are characterized by the quantum numbers l, s ($= ½$) and j respectively.

1) Find the expectation value $\langle\vec{L}\cdot\vec{S}\rangle$ in terms of l, s and j. (Hint: Begin by calculating the expectation value of $J^2 \equiv \vec{J}\cdot J$.)

2) Consider an electron with orbital angular momentum quantum number $l = 2$.
What are the possible values of j ?
What are the possible values of $\langle\vec{L}\cdot\vec{S}\rangle$?
What are the possible values of the projection quantum numbers m_l, m_s, m_j ?

b) The "2p shell" of an atom consists of all those electrons whose principal quantum number is $n = 2$ and whose orbital angular momentum quantum number is $l = 1$.
State the Pauli exclusion principle in the most general form you can, and use it to explain why the maximum number of electrons in the 2p shell is 6.

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Preliminary Comprehensive Examination

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THE EXAM WILL BEGIN AT 9:00 AM ON THURSDAY, OCTOBER 24, 2002 AND CONTINUE THRU 5:00 PM FRIDAY, OCTOBER 25, 2002.