Physics Department – The Catholic University of America

Preliminary Examination for Graduate Students

27 August 2010

This examination is designed to test your knowledge of undergraduate physics. Since you may not have covered all topics in your undergraduate courses, do not panic if you see something unfamiliar, but simply write down as much as you can and proceed to the next question.

The format of this examination is slightly different from that of the exams given in 2008 and earlier. In each of the four subject areas (classical mechanics, E&M, thermodynamics and quantum physics), there are three shorter questions instead of two longer ones.

You should count on spending no more than one hour on each subject area – it is more important to show what you know in all four areas than to excel in some and omit others.

Please begin each question on a new page.

Students are not permitted to bring any electronic devices (including cell phones, PDAs and calculators) into the examination room. Any such devices have to be handed over to the proctor.

Only calculators and mathematics tables provided by the department may be used.

Note: The symbols $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ always represent unit vectors in the $+x$, $+y$ and $+z$ directions respectively.

Part I. Classical Mechanics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

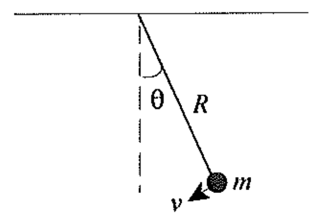
M1. A force in the xy plane is given by $\vec{F} = \frac{F_0}{r}(y\,\hat{i} - x\,\hat{j})$, where $r \equiv \sqrt{x^2+y^2}$ and F_0 is a constant.

a) Show that the magnitude of this force is F_0, and that its direction is perpendicular to $\vec{r} = x\,\hat{i} + y\,\hat{j}$.

b) Find the work done by this force on a particle that moves in a circle of radius R centered at the origin.

c) Is this force conservative? Explain your answer.

M2. A point mass m is attached to the end of a string of fixed length R which can swing freely in a vertical plane in the gravitational field of the Earth. The instantaneous speed of the mass is v, and the string makes angle θ with the vertical.



a) Label the forces on m and draw a free-body diagram.

b) Given θ and v, find the tension T in the string and the tangential acceleration at this instant.

c)* Write the Lagrangian for this system, using θ as the single generalized coordinate.

d)* Use the Lagrange equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$, to derive a differential equation for θ as a function of time.

*If you have not learned the Lagrangian formalism, then:
Use energy conservation to derive an expression for v as a function of θ, assuming that $v = v_0$ when $\theta = 0$. (You may assume that $-\pi/2 < \theta < \pi/2$.)

M3. A satellite of mass m orbits about a planet of mass $M \gg m$ under the influence of their mutual gravitational attraction. Since $M \gg m$, we can treat the planet as being at rest.

a) What is the shape of the orbit? (Newton solved this problem for us – just state the answer.)

The position of the satellite can be specified by polar coordinates r, θ with r measured from the center of the planet. For a non-circular orbit, r and $\dot{\theta}$ are not constant. In terms of m, r, θ, $\dot{r}$ and $\dot{\theta}$, write an expression for

b) the angular momentum of the satellite (about the planet). (It may be useful to start with writing the velocity and other vector quantities using unit vectors $\hat{r}$ and $\hat{\theta}$.)

c) the mechanical energy (kinetic + potential) of the satellite.

d) Since gravitation is a central force, the angular momentum of the satellite is constant. Explain why this statement is true. (Hint: consider the relation between torque and angular momentum.)

e) Use the constant angular momentum L to eliminate $\dot{\theta}$ from the energy, and write the total energy as a function of L, r and $\dot{r}$. (The part that depends on r is called the "effective potential energy.")

Part II. Thermodynamics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

T1. A sample of an ideal gas is subjected to adiabatic expansion from volume V to volume $V+dV$ (adiabatic means without transfer of thermal energy to or from the environment).

a) How much work has been done by the system?

b) The change in internal energy of the gas is $C_V\, dT$, in which dT is the change in temperature resulting from the expansion. How is this energy quantity related to the work done?

c) From this relation and the ideal gas equation, $P\,V = n\,R\,T$, derive a differential equation in V and T.

d) Solve this equation and introduce the initial conditions (V_0, P_0, T_0) to obtain the relation between V and T during adiabatic expansion or compression of a ideal gas.

e) With use of the ideal gas equation convert this equation into a relation between P and T, and show that $P/P_0 = (V_0/V)^\gamma$ with $\gamma = C_P/C_V$.

-2. A waterfall is 160 feet high. How much warmer is the water at the bottom than at the top, as the result of the conversion of potential energy into thermal energy? The energy required to raise the temperature of 1 kg of water by 1degree C is 4.184 kJ.

T3. A system can be in one of two states whose energies are E_1 and E_2. In thermal equilibrium at temperature T, find

a) the average energy of the system.

b) the ratio $\frac{N_2}{N_1}$ of the number of particles in state E_2 to the number of particles in state E_1.

Express your answers in terms of the energies, the temperature, and the Boltzmann constant.

Part III. Electricity and Magnetism

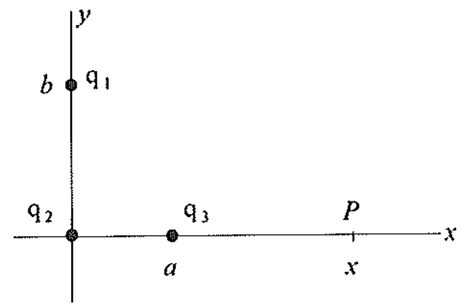
Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

E1. Three charges are located as shown:

q_1 at x=0, y=b

q_2 at x=0, y=0

q_3 at x=a, y=0



a) Write a complete expression for the electric field at point P on the x axis (position x, 0). (You need not simplify it.)

b) Write a complete expression for the electric potential at point P. (You need not simplify it.)

E2. Calculate the magnetic field at position z along the rotation axis (where $z = 0$ in the plane of the loop or disk) for each the following systems:

a) a circular current loop of radius r with constant current I.

b) a circular ring of radius r, carrying a uniform linear charge density λ, and rotating about its axis with angular velocity ω.

c) a disk of radius R with uniform surface charge density σ, rotating with angular velocity ω. (You need not solve the final integral.)

E3. An electromagnetic wave has a frequency of 100 MHz and is traveling in a vacuum. The magnetic field is given by $\vec{B}(z,t) = (10^{-8}\ T) \cos(kz - \omega t)\,\hat{i}$.

a) Find the wavelength and the direction of propagation of this wave.

b) Using the appropriate Maxwell equation(s), find the electric field vector. [Even if you know the answer without using Maxwell's equations, tell which of Maxwell's equations you would have to use.]

c) Calculate the Poynting vector, and find the intensity of this wave.

Part IV. Quantum Mechanics/Modern Physics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

Q1. The wave function for a particle in a particular potential has the form
$\Psi(x,t) = A \sin(\pi x)\, e^{-it} + B \sin(2\pi x)\, e^{-4it}$, where A and B are non-zero real numbers.

a) Find the distribution of **probability per unit length** as a function of x and t.

Find the probability per unit length (as a function of t) that the particle will be found
b) near $x = 0.25$
c) near $x = 0.5$

Q2. In coordinate space the raising and lowering operators for a harmonic oscillator are given by

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}\left(\mp\hbar\frac{d}{dx} + m\omega x\right)$$ (upper sign for $\hat{a}_{+}$, lower sign for $\hat{a}_{-}$).

a) Find the differential equation for the ground-state wave function $\psi_0(x)$ by applying the lowering operator to it: $\hat{a}_{-}\,\psi_0(x) = 0$.

b) Solve this equation to find $\psi_0(x)$. (You need not calculate the normalization constant.) [Hint: You can separate the equation by writing $d\psi_0/\psi_0$.]

c) Construct the wave function for the first excited state $\psi_1(x)$ by applying the raising operator to $\psi_0(x)$: $\psi_1(x) = N_1\,\hat{a}_{+}\,\psi_0(x)$ (where N_1 is some constant.)

d) (This part does not depend on the answers to the previous parts.)
If the a harmonic oscillator is initially in the state given by $\Psi(x,0) = \frac{1}{\sqrt{5}}[2\,\psi_0(x) + \psi_1(x)]$, where $\psi_0(x)$ and $\psi_1(x)$ are the normalized wave functions corresponding to the two lowest energy states, find the expectation value of the energy (in units of $\hbar\omega$). If you don't remember the energy formula for the harmonic oscillator, then write your answer in terms of the energies E_0 and E_1 corresponding to $\psi_0(x)$ and $\psi_1(x)$.

Q3. Consider the two electrons in an $l = 1$ shell of an atom (l is the orbital angular momentum quantum number for a single electron).
a) Find all possible values for the total **orbital** angular momentum quantum number l_{total} of these two electrons.
b) What are the possible **projections** L_z of this total orbital angular momentum along the z direction? (This should be a physical quantity in units of $\hbar$.)
c) Find all possible values for the total **spin** angular momentum quantum number s_{total} of these two electrons.
d) For each possible combination of l_{total} and s_{total}, find the possible values of the total angular momentum quantum number j_{total} (representing the sum of spin and orbital angular momentum).