Physics Department – The Catholic University of America

Preliminary Examination for Graduate Students

28 August 2009

This examination is designed to test your knowledge of undergraduate physics. Since you may not have covered all topics in your undergraduate courses, do not panic if you see something unfamiliar, but simply write down as much as you can and proceed to the next question.

The format of this examination is slightly different from that of previous years. In each of the four subject areas (classical mechanics, E&M, thermodynamics and quantum physics), there are three shorter questions instead of two longer ones.

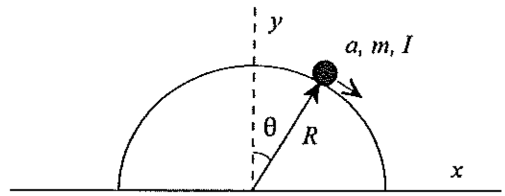
You should count on spending no more than one hour on each subject area – it is more important to show what you know in all four areas than to excel in some and omit others.

Please begin each question on a new page. Only calculators and mathematics tables provided by the department may be used.

Part I. Classical Mechanics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

M1. A spherical ball of radius a, mass m and moment of inertia I rolls (without slipping) on the exterior of a hemispherical hill of radius R whose center is at the origin. Let θ be the angle from the y axis to the position of the ball.



Write

a) the potential energy of the ball as a function of θ.

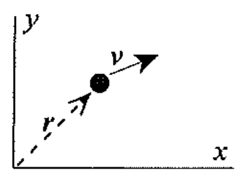
b) the kinetic energy of the ball (translational plus rotational) as a function of $\dot{\theta} = \frac{d\theta}{dt}$.

M2. a) Write the Lagrangian function for the system of Problem 1, with θ as the generalized coordinate.

b) Use the Lagrange equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$ to derive a differential equation for θ as a function of time.

[If you do not know the Lagrange formalism, find a differential equation for θ in some other way, e.g. by taking components of $F = ma$ along the surface of the hemisphere.]

M3. A pointlike projectile of mass m is fired upwards from the origin. At any time after launch, the position and velocity of the projectile are described by the variables x, y, $\dot{x}$ and $\dot{y}$, or alternatively by the vectors $\vec{r} = \hat{i}x + \hat{j}y$ and $\vec{v} = \hat{i}\dot{x} + \hat{j}\dot{y}$. Neglect air resistance.



In terms of x, y, $\dot{x}$, $\dot{y}$, and the Cartesian unit vectors $\hat{i}, \hat{j}, \hat{k}$,

a) Write the vector angular momentum $\vec{L}$ of the projectile about the origin. (Hint: Calculate the cross product using components.)

b) Write the vector torque $\vec{\tau}$ about the origin exerted on the projectile by gravity.

c) Write the relationship between the angular momentum of a particle and the torque on a particle, and solve for $\ddot{y}$. Show that it leads to the expected result if $\ddot{x} = 0$.

Part II. Thermodynamics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

T1. State in words the difference between intensive and extensive variables in thermodynamics. Give two examples of each.

T2. One mole of an ideal gas is taken from (T_1, V_1) to (T_2, V_2), where T is the temperature and V is the volume. Starting from the first and second laws of thermodynamics show that the change in entropy is:

$$\Delta S = C_V \ln\left[\frac{T_2}{T_1}\right] + R \ln\left[\frac{V_2}{V_1}\right].$$

T3. Helium gas (γ=1.67) is initially at a pressure of 16 atm, a volume of 1 L, and a temperature of 600 K. It is expanded isothermally until its volume is 4 L and is then compressed at constant pressure until its volume and temperature are such that an adiabatic compression will return the gas to its original state.

a) Sketch this cycle on a PV diagram.
b) Find the volume and temperature after the isobaric compression.
c) Find the work done during each cycle.

Part III. Electricity and Magnetism

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

E1. a) Write Maxwell's equations in integral form. Give the familiar name of each one (e.g. Gauss's law), if applicable.

b) For each equation in part (a), write the corresponding equation in differential form.

E2. A plane electromagnetic wave in a medium is described by

$$E_x(z,t) = 20\sin(10z - 2.0\times10^9\, t), \quad E_y = E_z = 0$$

where the constants have SI units such that E is given in V/m, z in meters and t in seconds.

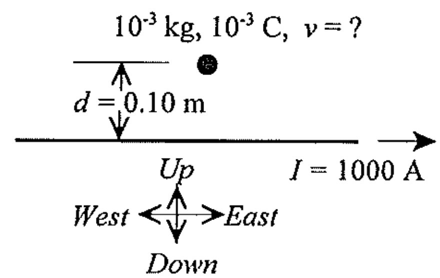
a) What is the direction of propagation of the wave?

b) Find the wavelength and the frequency of the wave.

c) Find the **speed of propagation** of the wave, and the **index of refraction** of the medium.

d) What can you say about the magnitude and direction of the **magnetic field** of this wave?

E3. A long, straight horizontal wire carries a current of 1000 A from west to east. A charged particle of mass 10^{-3} kg and charge $+10^{-3}$ C is 0.10 m directly above the wire, and is moving in a horizontal plane.



a) What is the direction and magnitude of the **magnetic field** of the wire at the position of the particle? (Neglect the magnetic field of the earth.)
$\frac{\mu_0}{4\pi} = 1.00\times10^{-7}$ H/m

b) Find a **direction of motion** of the particle such that the magnetic force of the wire on the particle is exactly vertically upward when the particle is directly above the wire.

c) What is the the **velocity** of the particle if the magnetic force of the wire exactly balances the gravitational force on the particle when it is directly above the wire?

d) Are your answers to (b) and (c) unique? Explain.

Part IV. Quantum Mechanics/Modern Physics

Answer all three questions, but do not take more than 1 hour (until you have worked on the other parts of the exam.)

Q1. **Define** each of the following, and give an **example** where appropriate.
Let operators be represented by the symbols $\hat{A}, \hat{B},\ldots$, and state functions (or wave functions) by the symbols ψ_n or $|n\rangle$.

a) Eigenvalue equation, eigenvalue, eigenfunction
b) Normalized wave function
c) Expectation value of a variable
d) Stationary state
e) Orthogonal wave functions

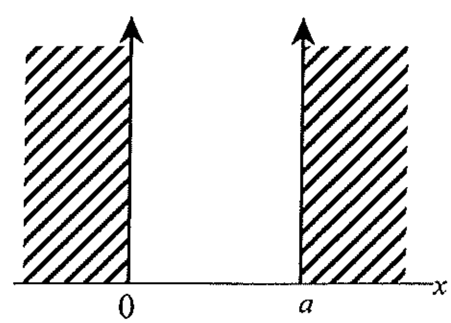
Q2. (Part 1)
For a single electron of orbital angular momentum quantum number $l = 3$, write
a) the expectation value of L^2 (orbital angular momentum squared).
b) the possible observable values of L_z (the z-component of orbital angular momentum).

(Part 2)
The total orbital angular momentum of two particles is defined classically as $\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2$. Similarly the total spin is defined as $\vec{S}_{tot} = \vec{S}_1 + \vec{S}_2$, and the total angular momentum (spin and orbital) is defined as $\vec{J}_{tot} = \vec{J}_1 + \vec{J}_2 = \vec{L}_{tot} + \vec{S}_{tot}$. In quantum mechanics, this "addition" is performed using the rules for combining angular momentum quantum numbers.

Two electrons (spin quantum numbers $s_1 = s_2 = ½$) in a given atom have orbital angular momentum quantum numbers $l_1 = 2$ and $l_2 = 3$. Find all possible values of their
c) total orbital quantum number l_{tot}
d) total spin quantum number s_{tot}
e) total angular momentum quantum number j_{tot}

Q3. A one-dimensional infinite square well extends from x=0 to x =a:

$$V(x) = \infty \quad x < 0, \ x > a$$
$$= 0 \quad 0 < x < a$$



a) Write the time-independent Schrödinger equation for a particle of mass m in this potential, and show that the solutions inside the well are given by

$$\psi(x) = A_n \sin(k_n x), \quad n = 1, 2, 3, \ldots.$$

b) Find the constants A_n and k_n. Describe the physical reasoning used in determining them.
c) Find the energy E_n of the particle.