Preliminary Examination **Physics Department** **27 October 2007**

You may use mathematical tables and non-programmable calculators provided by the exam proctor.

Classical Mechanics

1. A simple pendulum consists of a point mass M suspended from a massless string of length L.

   a) Write the Lagrangian function for this system in terms of θ, the angle which the string makes with the vertical. (Let $\theta = 0$ when the mass is at its lowest point.)

   b) Use Lagrange's equations to find the equation of motion (a differential equation for θ.)

   c) Using small-angle approximation (valid for $\theta \ll 1$ radian), find the frequency of oscillation of the pendulum.

(If you have no experience with Lagrange's equations, find the frequency of small oscillations of the pendulum by another method.)

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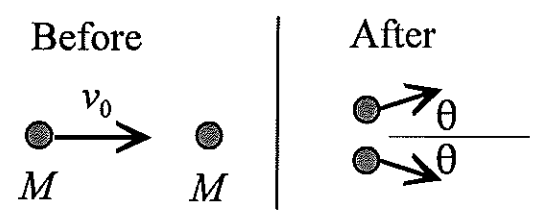
2. A particle of mass M moving at initial velocity v_0 collides with a second particle of the same mass M which is initially at rest. After the collision, it is observed that both particles are moving at *equal angles* θ relative to the original direction of motion, as shown in the figure. Do not assume the collision is elastic.



   a) Let the speeds of the two particles after collision be v_1 and v_2. Using an appropriate conservation law, prove that $v_1 = v_2$.

   b) Find the speed of the particles after the collision.

   c) Find the total kinetic energy of the system after the collision, and compare it to the total kinetic energy before the collision.

   d) What would the angle θ be if the collision is *elastic*?

Electricity and Magnetism

3. A solid sphere of radius R has a non-uniform charge density $\rho(r) = Cr^2$ (where C is a constant.)

a) Find the electric field $E(r)$ at points inside the sphere ($r < R$).

b) Find the electric field $E(r)$ at points outside the sphere ($r > R$).

c) Find the potential $V(r)$ at points outside the sphere ($r > R$).

d) Find the potential $V(r)$ at points inside the sphere ($r < R$).

e) Sketch (qualitatively) a graph of $V(r)$ versus r.

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4. a) Write the four Maxwell equations in differential form, in the presence of charge density ρ and current density $\vec{J}$, in free space ($\varepsilon=\varepsilon_0$, $\mu=\mu_0$). Also write each equation in integral form, and give its usual name (if any.)

b) Now assume that the charge density and current density in a region are both 0. Starting with Maxwell's equations in differential form, and looking up vector calculus identities as needed, show that the electric field in free space obeys a **wave equation**, and show explicitly from your derivation how the speed of the wave depends on the constants ε_0 and μ_0. (Hint: Begin by taking the curl of both sides of suitable Maxwell equations.)

Thermodynamics

5. For carbon dioxide (CO_2) gas, the constants in the van der Waals equation

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

are $a = 0.364\ \text{J}\cdot\text{m}^3/\text{mol}^2$ and $b = 4.27 \times 10^{-5}\ \text{m}^3/\text{mol}$. The ideal gas constant is given by $R = 8.31\ \text{J/mol}\cdot\text{K} = 0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K}$, and $1\ \text{atm} = 1.01\times10^5\ \text{Pa}$.

a) If 1.00 mol of CO_2 gas at 350 K is confined to a volume of $400\ \text{cm}^3$, find the pressure of the gas (in reasonable units) using (1) the ideal gas equation and (2) the van der Waals equation.

b) Which equation gives a lower pressure? What is the percentage difference of the van der Waals result from the ideal gas result?

c) Explain qualitatively the physical meaning of the constants a and b.

d) If the volume expands to $4000\ \text{cm}^3$, do you expect the percentage difference to increase or decrease? Explain briefly.

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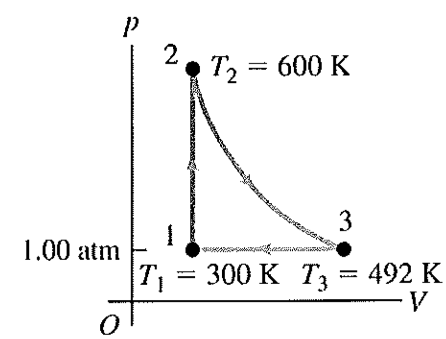
6. A heat engine takes 0.350 mol of a diatomic ideal gas around the cycle shown in the pV-diagram. Process $1\rightarrow2$ is at constant volume, process $2\rightarrow3$ is adiabatic, and process $3\rightarrow1$ is at a constant pressure of 1.0 atm. ($1\ \text{atm} = 1.01\times10^5\ \text{Pa}$.)
For a diatomic ideal gas, $C_V = 5/2\ R$ and $\gamma = C_p/C_V = 1.40$.



a) Find the pressure and volume at points 1, 2, and 3.

b) Calculate the heat transfer Q, the work W, and the change in internal energy ΔU for each of the three processes.

c) Find the net work done by the gas in the cycle.

d) Find the net heat flow into the engine in one cycle.

e) Find the thermal efficiency of the engine.

Quantum Mechanics/Modern Physics

7. Ultraviolet light from a given source shines on a platinum cathode. It is observed that electrons are emitted from the cathode by means of the photoelectric effect. According to the Handbook of Chemistry and Physics, platinum has a work function of 5.65 eV.

Useful constants: $hc = 1240$ eV·nm, $e = 1.602\times10^{-19}$ C.

a) From the information given, determine the minimum or maximum wavelength of the UV light.

b) Now a reverse potential difference is applied to the cathode. It is found that a retarding potential of 3.0 volts is sufficient to stop the photoelectric emission of electrons from the cathode. Find the wavelength of the UV light.

c) If a steady current of 1.0 microampere (10^{-6}A) is measured flowing from the photocathode, how many electrons per second are being emitted? Does the answer depend on the wavelength of the light?

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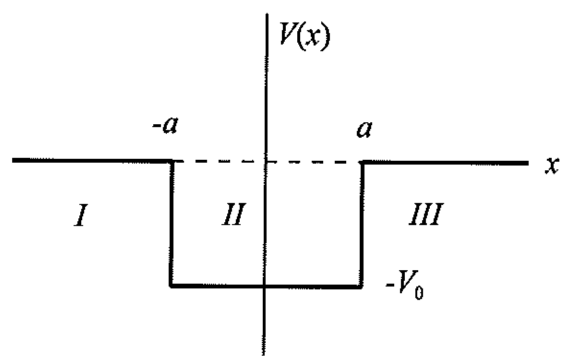
8. A particle of mass m is bound in a finite one-dimensional square well described by the potential energy function

$$\begin{aligned} V(x) &= 0 \qquad x<-a \qquad \text{(Region I)} \\ &= -V_0 \quad -a\le x\le a \quad \text{(Region II)} \quad (V_0>0). \\ &= 0 \qquad x>a \qquad \text{(Region III)} \end{aligned}$$



The energy of the particle has some definite negative value E_1, where $-V_0<E_1<0$.

a) In each of the 3 regions, write
   (1) the time-independent Schrödinger equation, and
   (2) the general form of the normalizable spatial wave function $\psi(x)$ of the particle, assuming that $\psi(x)$ is an even function of x (i.e. $\psi(-x) = \psi(x)$). Calculate the values of all the constants in your function that are determined by the given quantities.

b) Make a qualitative sketch of $\psi(x)$ versus x.

c) Apply the boundary conditions at $x=-a$ and $x=a$ to write a set of equations which can (in principle) be solved to find the unknown constants in the wave function. (Do not solve.)

d) Write the time-dependent wave function $\Psi(x,t)$ in terms of $\psi(x)$. (Do not substitute for $\psi(x)$.)