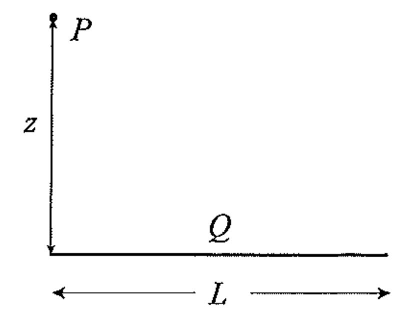
Physics Preliminary Exam
August 2004

Electricity and Magnetism

EM-1. Calculate the electrostatic potential $V(z)$ at a point P a distance z directly above the left end of a line charge of length L and total charge Q.



HINT: Use $V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}$.

EM-2 The magnetic field inside a long hollow solenoid of radius R carrying a current

$$i = I_0 \cos\omega t$$

is given by

$$B = \mu_0 n i = \mu_0 n I_0 \cos\omega t,$$

where n is the number of turns of wire per unit length in the solenoid. Calculate the electric field inside the solenoid induced by this time-varying magnetic field as a function of the distance from the solenoid axis $r < R$.

HINT: You may find it useful to recall Maxwell's equations in their integral form:

$$\oint_S \vec{E}\cdot d\vec{A} = q/\epsilon_0 \qquad \oint_S \vec{B}\cdot d\vec{A} = 0$$

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A} \qquad \oint_C \vec{B}\cdot d\vec{s} = \mu_0 i + \mu_0\epsilon_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

Mechanics

M-1. A particle of mass m moves under the influence of an central force described by a potential energy function $V(r)$ where $r = |\vec{r}|$ and $\vec{r}$ is the position vector from the force center to the particle.

(a) Show that the angular momentum of the particle $\vec{l} = \vec{r} \times \vec{p}$ is conserved. Here $\vec{p}$ is the particle's momentum.

(b) Using plane polar coordinates r and ϑ, write expressions for the particle's kinetic energy $\frac{1}{2} m v^2$ and angular momentum $l = |\vec{l}|$. Substitute the expression for l into the kinetic energy equation to eliminate $\dot{\vartheta}$.

(c) Use this last expression to write an equation for the system's energy $E = \frac{1}{2} m v^2 + V$. Identify the equivalent one-dimensional potential $V'(r)$.

(d) Consider explicitly the attractive Coulomb potential energy function $V(r) = -\dfrac{C}{r}$, where C is a positive constant. For a given angular momentum l, obtain an expression for the radius of the stable circular orbit.

M-2. Newton's law for a harmonic oscillator with damping proportional to the velocity is of the form

$$m \frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt},$$

where m, k, and b are the mass, stiffness and damping coefficient respectively.

(a) Show explicitly that solutions to this can be obtained in the form

$$x(t) = C_1 e^{-\gamma t} e^{-i\omega t} + C_2 e^{-\gamma t} e^{+i\omega t},$$

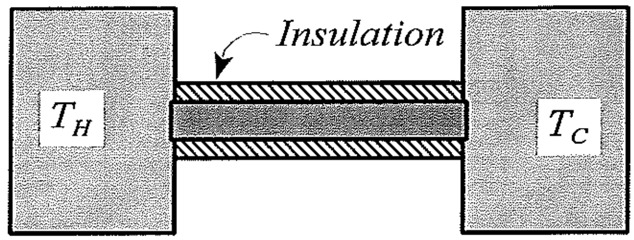
where C_1, C_2, γ and ω are constants. Express γ and ω in term of the system parameters m, k, and b.

(b) Distinguish the underdamped and overdamped cases in terms of the relative values of γ and ω and sketch x versus t for each of these cases.

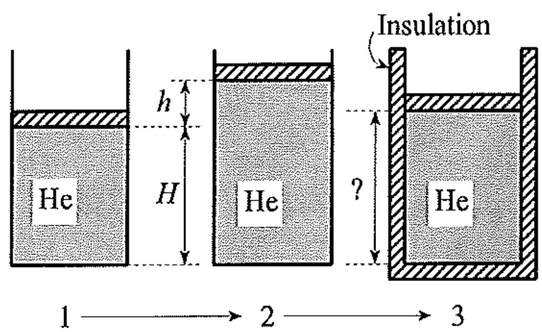
(c) What is meant by critical damping?

Thermodynamics/Statistical Physics

TH-1 An insulated container is filled with a mixture of water and ice at $T_C = 0°C$. Another container is filled with water that is continuously boiling at $T_H = 100°C$. In a series of experiments, the containers are connected by various thick rods that pass through the walls of the containers as shown in the diagram. The rod is insulated in such a way that no heat is lost to the surroundings. In experiment 1, a copper rod is used, and the ice melts in a time $t_1 = 20$ minutes. In experiment 2, a steel rod is used and the ice melts in a time $t_2 = 60$ minutes. How long would it take the ice to melt if the two rods are used end to end, that is "in series?"



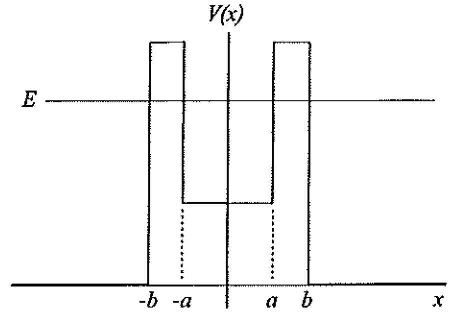
TH-2 Monatomic helium gas in a vertical cylindrical container is in thermal and mechanical equilibrium with the surroundings. The gas is confined by a movable heavy insulating piston. Initially (1), the height of gas in the cylinder is H. The piston is slowly elevated a distance h from its equilibrium position and then kept in the elevated position (2) long enough for thermodynamic equilibrium to be reestablished. After that the container is insulated and the piston is released. After the piston comes to rest (3), what is its new equilibrium position?



HINT: Assume that the helium can be treated as an ideal gas.

Modern Physics/Quantum Mechanics

QM-1 Consider a particle of mass m moving under the influence of the one-dimensional potential sketched at the right. The particle has energy E as indicated by the horizontal line on the diagram. Sketch the wavefunction $\psi(x)$ of the particle for $x > 0$. You need not solve the Schroedinger equation; the qualitatively correct shape of ψ is all that is required. Assume that $E >> h^2/2ma^2$, where h is Planck's constant.

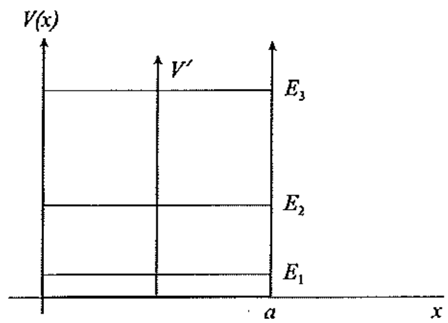


QM-2 At the right is sketched a one-dimensional infinite square well of width a.
Also sketched are the three lowest energy levels. In general, the energy eigenvalues for this potential are given by

$$E_n = \frac{(\hbar k_n)^2}{2m} \qquad (n = 1, 2, 3 \ldots)$$

and the corresponding eigenfunctions are

$$\psi(x) = \sqrt{2/a}\,\sin k_n x\,.$$



Suppose that a perturbing delta-function potential

$$V' = \varepsilon a\,\delta(x - a/2)$$

is added as shown on the figure. Calculate the first-order correction to the ground-state energy of the system.

HINT: Recall that $\Delta E_n = \int dx\; \psi_n^*\, V'\, \psi_n$.