Physics Preliminary Exam
August 2003

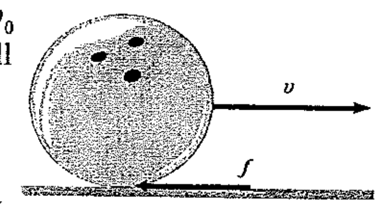
Mechanics

M-1. Small spherical particles falling through the air experience a viscous drag force given by Stokes' law $f = 6\pi \eta r v$ where r is the radius of the particle, v is its instantaneous velocity, and η is the viscosity of the air.

(a) Obtain an expression for the terminal velocity of such a particle.

(b) Estimate the time that it would take for a pollution particle of radius 10^{-5} m and density 2000 kg/m^3 to fall from a 100-m tall smokestack. Take $\eta = 1.8 \times 10^{-5}$ N$\cdot$s/m^2 and assume that the air is still.

M-2. A bowling ball of mass M and radius R is thrown such that at the instant it touches the floor, it is moving horizontally with a speed v_0 and is not rotating. The coefficient of kinetic friction between the ball and the floor is μ. Obtain expressions for

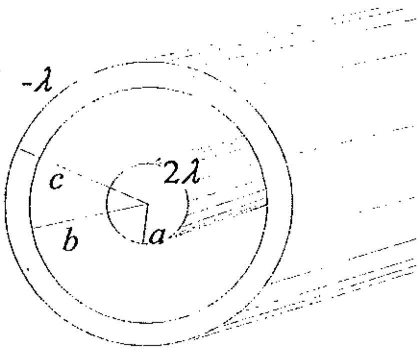


(a) the time that the ball slides before it rolls without slipping and

(b) the distance that the ball slides before it rolls without slipping.

Electricity and Magnetism

EM-1. A long solid ***nonconducting*** cylinder of radius a is placed coaxially with a long ***conducting*** cylindrical shell of inner radius b and outer radius c. A charge per unit length $+2\lambda$ is distributed uniformly throughout the solid cylinder and charge per unit length, $-\lambda$, is placed on the outer shell as sketched at the right.



- *(a)* Obtain the magnitude of the electric field at a point a distance r from the axis if $r \le a$, *i.e.*, inside the solid cylinder.
- *(b)* Obtain the magnitude of the field at a point a distance r from the axis if $a \le r \le b$, *i.e.*, in the space between the cylinders.
- *(c)* Obtain the magnitude of the electric field at a point a distance r from the axis, where now $r \ge c$.

EM-2 The intensity (average power per unite area) of sunlight striking the upper atmosphere is 1.4 kW/m^2.

- *(a)* Calculate the rms values of the electric field E and the magnetic field B due to the sun at the upper atmosphere of the earth.
- *(b)* What is the average power output of the sun?
- *(c)* What is the intensity at the surface of the sun?

DATA: $\epsilon_0 = 8.854 \times 10^{-12}\, C^2/N\cdot m^2$
$\mu_0 = 4\pi \times 10^{-7}\, N/A^2$
Sun- earth distance = 1.5×10^{11} m
Solar radius = 6.96×10^8 m

Thermodynamics/Statistical Physics

TH-1 A small pond has a layer of ice of thickness s floating on its surface at some instant. Suppose the air temperature is lower than the freezing point of water by an amount ΔT.

(a) Obtain an expression for the rate at which the thickness of the ice increases. The density and thermal conductivity of ice are ρ and k respectively. The latent heat of fusion of water is L.

(b) If d is initially 0.010 m and $\Delta T = 20°C$, how long will it take for the thickness of the ice layer to double? Use $\rho = 917\ \text{kg/m}^3$, $k = 0.592$ W/m·deg and $L = 333.5$ kJ/kg.

TH-2 The behavior of a gasoline engine can be approximated by the so-called *Otto cycle* for an ideal gas sketched at the right.

(a) Compute the heat input Q_i and the heat output Q_o for this cycle

and

(b) show that the efficiency

$$\eta \equiv W/Q_i = (Q_i - Q_o)/Q_i$$

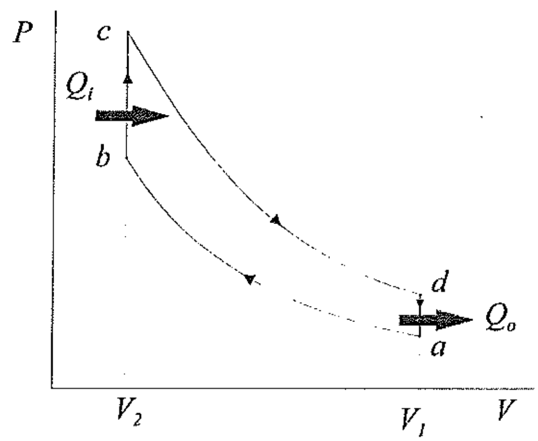
is given by

$$\eta = 1 - \frac{T_d - T_a}{T_c - T_b},$$

where T_a is the temperature of state a, and so on.

where $V_1 = V_a = V_d$ and $V_2 = V_b = V_c$.

HINT: For an ideal gas, the specific heat of an ideal gas at constant volume is a constant, say C_v.



Otto cycle, representing the internal combustion engine. The gasoline-air mixture enters at a and is adiabatically compressed to b. It is then heated at constant volume by ignition from the spark to c. The power stroke is represented by the adiabtic expansion from c to d. The cooling from d to a represents the exhausting of the burned gases and the intake of a fresh gasoline-air mixture.

Physics Preliminary Exam
August 2003

Modern Physics/Quantum Mechanics

QM-1 For a simple harmonic oscillator in its lowest energy state, the following are true

(1) $<p> = 0$;

(2) $<x> = 0$; and

(3) <kinetic energy> = <potential energy>, that is

$$<p^2>/2m = m\omega^2<x^2>/2.$$

Use the uncertainty principle in the form

$$\Delta x \Delta p \approx \hbar/2$$

to estimate the ground state energy of the oscillator.

In case you've forgotten,

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

and

$$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2.$$

QM-2 In the process of pair production, an electron-positron pair are created spontaneously by the annihilation of a gamma ray near a nucleus. In the center-of-mass frame of reference, the two particles fly off in opposite directions at equal speeds of $0.75\,c$.

(a) Calculate the speed of the positron in the frame in which the electron is at rest.

(b) Calculate the momentum of the positron in this reference frame. Compare it with the positron's momentum in the center-of-mass frame.

(c) Calculate the energy and wavelength of the gamma ray, assuming that the nearby nucleus is infinitely massive.

DATA: Electron rest mass, $m_0 = 0.511\ \text{MeV}/c^2$
$hc = 1.24 \times 10^{-6}\ \text{eV}\cdot\text{m}$

If you can't remember the relativistic "velocity addition law," you can ask the proctor. This will reduce your maximum score on this problem from 100 to 80.