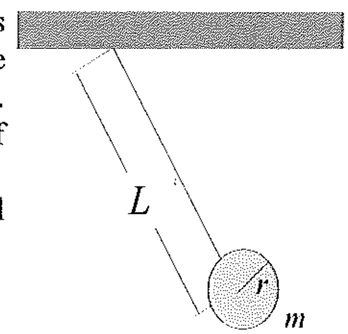
Mechanics

M-1. The great limestone caverns were formed by dripping water. In this problem consider 0.10 - cm^3 water droplets that fall from a height of 5.0 m at a rate of 10 per minute.

(a) What is the average force exerted by the water drops on the limestone floor.
(b) Compare this with the weight of a water droplet.
(c) Estimate the force exerted on the cavern floor by a single water droplet.
(d) Explain why the result in *(c)* differs from your answers to parts *(a)* and *(b)*.

Make any reasonable assumptions that you feel are necessary to answer these questions, but state them clearly. Take the density of water to be 10^3 kg/m^3.

M-2. Consider a *physical* pendulum that consists of a spherical bob of radius r and mass m suspended from a string as in the figure at the right. The distance from the center of the sphere to the point of support is L. When $r < L$, such a pendulum can be treated as a simple pendulum of length L.



(a) Show that the period of the physical pendulum for small oscillations is

$$T = T_0 \sqrt{1 + \frac{2r^2}{5L^2}} \, ,$$

where $T_0 = 2\pi\sqrt{L/g}$ is the period of a simple pendulum of length L.

(b) Show that, when r is small compared to L, the period is given approximately by

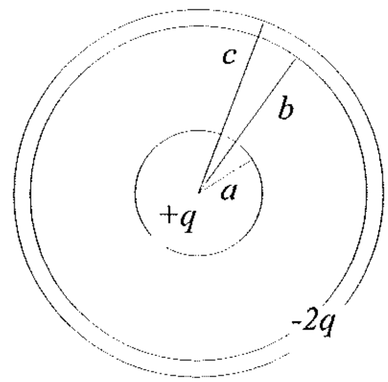
$$T \approx T_0 (1 + r^2/5L^2).$$

(c) For $L = 1$ m and $r = 2$ cm, find the relative error if the approximation $T = T_0$ is used.

(d) How large must the radius of the sphere be for the error to be 1%?

Electricity and Magnetism

EM-1. A solid ***nonconducting*** sphere of radius a is placed at the center of a ***conducting*** spherical shell of inner radius b and outer radius c. An amount of charge $+q$ is distributed uniformly throughout the solid sphere and an amount of charge, $-2\,q$, is placed on the shell as sketched at the right.

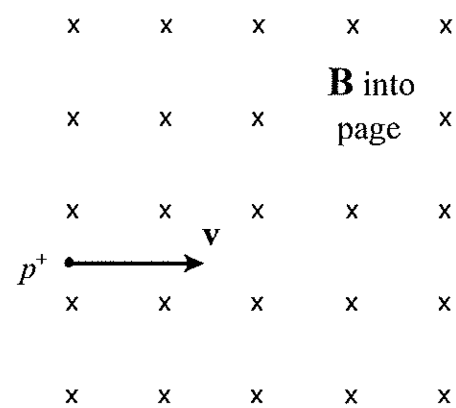


(a) Obtain the magnitude of the electric field at a point a distance r from the center if $r \le a$, *i.e.*, inside the small solid sphere.

(b) Obtain the magnitude of the field at a point a distance r from the center if $a \le r \le b$, *i.e.*, in the space between the spheres.

(c) Obtain the magnitude of the electric field at a point a distance r from the center, where now $r \ge c$.

EM-2 A proton (mass, $m = 1.67 \times 10^{-27}$ kg, charge $= e = 1.60 \times 10^{-19}$ C) with kinetic energy, $K = 10$ MeV (1.0 eV $= 1.60 \times 10^{-19}$ J), enters a region where there is a uniform magnetic $B = 500$ mT directed perpendicular to the proton's velocity as shown in the sketch.



Describe as completely and as quantitatively as you can the trajectory of the proton in the region of non-zero magnetic field. Make a rough sketch of this trajectory.

DATA: For a proton $mc^2 \approx 938$ MeV.

Thermodynamics/Statistical Physics

TH-1 The Einstein model of a crystalline solid gives the following formula for the molar heat capacity at constant volume:

$$c_v = 3R\left(\frac{T_E}{T}\right)^2 \frac{e^{T_E/T}}{(e^{T_E/T} - 1)^2},$$

where R is the universal gas constant, T is the temperature (in kelvins), and T_E is called the Einstein temperature (also in kelvins) and is a parameter that characterizes the solid.

(a) Show that at high temperatures ($T \gg T_E$) this formula reduces to the Dulong-Petit law $c_v = 3R$.

(b) Obtain the low temperature ($T \ll T_E$) limit Einstein specific heat formula. Does this reproduce the observed low temperature $c_v \propto T^3$ behavior?

TH-2 The behavior of a gasoline engine can be approximated by the so-called *Otto cycle* for an ideal gas sketched at the right.

(a) Compute the heat input Q_i and the heat output Q_o for this cycle and from this show that the efficiency $\eta \equiv W/Q_i = (Q_i - Q_o)/Q_i$ is given by

$$\eta = 1 - \frac{T_d - T_a}{T_c - T_b},$$

where T_a is the temperature of state a, and so on.

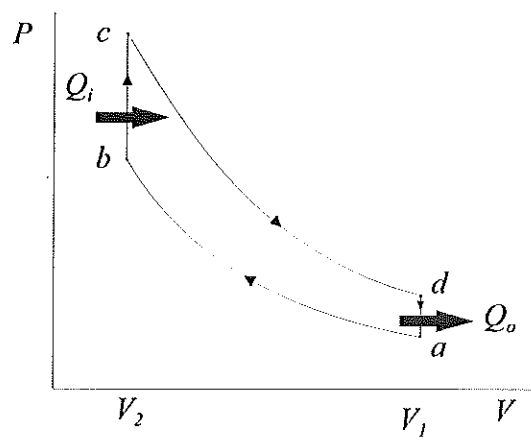
(b) Using the formula for the adiabatic expansion or compression of an ideal gas

$$T V^{\gamma - 1} = constant,$$

show that

$$\eta = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1},$$

where $V_1 = V_a = V_d$ and $V_2 = V_b = V_c$.



Otto cycle, representing the internal combustion engine. The gasoline-air mixture enters at *a* and is adiabatically compressed to *b*. It is then heated at constant volume by ignition from the spark to *c*. The power stroke is represented by the adiabtic expansion from *c* to *d*. The cooling from *d* to *a* represents the exhausting of the burned gases and the intake of a fresh gasoline-air mixture.

HINT: For an ideal gas, the specific heat of an ideal gas at constant volume is a constant, say C_v.

Physics Preliminary Exam
August 2002

Modern Physics/Quantum Mechanics

QM-1 An important consequence of the uncertainty principle is that a particle confined to some region of space has a minimum kinetic energy that is greater than zero. If Δp is taken to be the standard deviation in the momentum, it is given by

$$(\Delta p)^2 = \langle (p - \langle p \rangle)^2 \rangle,$$

where $\langle \ldots \rangle$ denotes the expectation value.

(a) Show that this is equivalent to

$$(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2.$$

Consider a particle of mass m moving in a one-dimensional box of length L such that $\langle p \rangle = 0$. Its average kinetic energy is then $\langle K \rangle = \langle p^2 \rangle / 2m$.

(b) By using the fact that Δx cannot be larger than L, show that its average kinetic energy cannot be smaller than $\hbar^2/8mL^2$.

In case you've forgotten, $\Delta x \, \Delta p \geq \hbar/2$.

QM-2 An electron in a hydrogen atom is in an eigenstate described by a wave function with principal quantum number n, orbital angular momentum quantum number ℓ, and z-component of orbital angular momentum quantum number m. Recall that the electron also has spin angular momentum described by the quantum numbers $s = ½$ and $m_s = \pm ½$. The total angular momentum of the electron **J** is thus the sum of the orbital and spin angular momenta, **L** and **S,** respectively, i.e.,:

$$\mathbf{J} = \mathbf{L} + \mathbf{S}.$$

What are the possible values for the angular momentum operators L^2, L_z, S^2, S_z, J^2, and J_z? Take $\ell = 2$. Give your answers in your exam booklet by constructing a table such as the one below.

Operator	Possible values
L^2	
L_z	
S^2	
S_z	
J^2	
J_z	