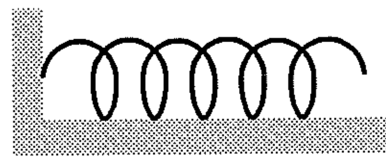
Physics Preliminary Exam
August 2001

Mechanics

M-1. A uniformly coiled spring of mass M (assume constant mass density along its length) has one end attached to a fixed support. The other end is given a small displacement and then released. Show that the frequency of oscillation is the same as that of a massless spring (with the same spring constant) with a mass m at its free end, and find that mass, *i.e.*, calculate m. Ignore friction.
HINT: Assume the displacement varies linearly along the spring and compute the kinetic energy of the spring.



M-2. In Yukawa's theory of nuclear forces, the attractive force between a neutron and a proton is given in terms of the potential

$$V(r) = -V_0 \frac{e^{-\alpha r}}{\alpha r} . \qquad (\alpha,\ V_0 = constants > 0)$$

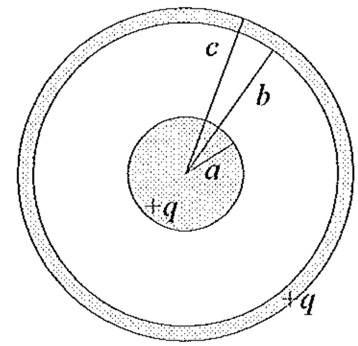
(a) Discuss the types of motion that can occur if a particle of mass m moves under the action of such a force (ignore the case of straight line, zero angular momentum motion).

(b) Calculate the angular momentum and total energy for motion in a circle of radius R.

(c) Calculate the period of for such motion.

(d) If a particle, moving in such a stable circular orbit, is slightly perturbed along the radial direction, it will execute small radial oscillations about the circular path. How would you go about calculating the frequency of such radial oscillations? *You need not carry out the calculation; just outline the steps you would follow in doing it.*

Electricity and Magnetism



EM-1. A solid ***nonconducting*** sphere of radius a is placed at the center of a ***conducting*** spherical shell of inner radius b and outer radius c. An amount of charge $+q$ is distributed uniformly throughout the solid sphere and an equal amount of charge, $+q$, is placed on the shell as sketched at the right.

(a) Obtain the magnitude of the electric field at a point a distance r from the center if $r \leq a$, *i.e.*, inside the small solid sphere.

(b) Obtain the magnitude of the field at a point a distance r from the center if $a \leq r \leq b$, *i.e.*, in the space between the spheres.

(c) Obtain the magnitude of the electric field at a point a distance r from the center, where now $r \geq c$.

(d) In terms of the given parameters (q, a, b, c) and fundamental constants, write expressions for the surface charge density (charge per unit area) on the inner surface of the spherical shell and on the outer surface of the spherical shell.

EM-2 Maxwell's equations in free space (in SI units) are

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

(a) Show that the field vectors are given by

$$\mathbf{B} = \nabla \times \mathbf{A} \text{ and } \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

(b) In the *Lorentz* gauge where

$$\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}.$$

show that both $\mathbf{A}$ and ϕ satisfy wave equations

$$\nabla^2 \mathbf{A} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} \text{ and } \nabla^2 \phi = \mu_0 \epsilon_0 \frac{\partial^2 \phi}{\partial t^2}.$$

<u>Thermodynamics/Statistical Physics</u>

TH-1 Consider a system composed of molecules all having a mass m in equilibrium at temperature T. The Maxwell-Boltzmann distribution giving the fraction of molecules with speed in the range v to $v + dv$ is

$$f(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}\,dv,$$

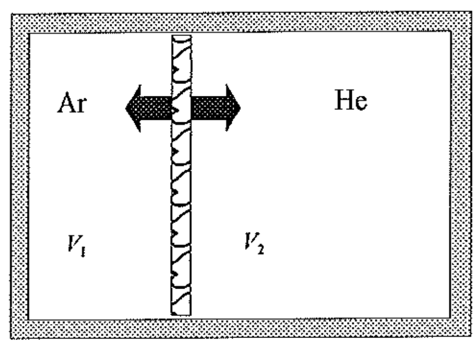
where k is Botzmann's constant. Use this to calculate

(a) the average speed of a molecule

and

(b) the average kinetic energy of a molecule.

TH-2 An insulated container of total volume V is divided into two parts of volumes V_1 and V_2 by a thermally conducting partition that is free to move as indicated by the arrows in the figure. The volume V_1 is filled with n moles of argon gas; V_2 contains the same number of moles of helium. The system is allowed to come to thermal and mechanical equilibrium. Assume that the densities are low enough that the gases can be treated as ideal.



(a) How do the volumes V_1 and V_2 compare?

(b) Suppose the temperature and pressure of the argon gas are T_1 and P_1, respectively. Are the temperature and pressure of the helium necessarily the same?

(c) Explain your answers to parts *(a)* and *(b)*.

(d) At a certain instant the partition is replaced by a rigid semipermeable membrane that allows helium atoms to flow into V_1, but does not allow the passage of argon atoms across it. What force per unit area must be exerted on the membrane to prevent it from moving?

(e) At a later instant the membrane is removed entirely. When the system comes to equilibrium, its entropy is different from it value prior to the partition being replaced. Calculate this entropy difference and explain its origin in physical terms.

Physics Preliminary Exam
August 2001

Modern Physics/Quantum Mechanics

QM-1 Consider a ball bouncing on a horizontal floor in a uniform gravitational field, *i.e.*, the acceleration due to gravity is a constant, g. Now, imagine the "ball" is an isolated atom of mass m bouncing against gravity, with perfect reflection at an idealized, perfectly flat, horizontal surface at $x = 0$.

(a) Draw the potential energy curve for this case, and beneath it sketch the approximate form that ψ must have for each of the lowest two energy levels. Indicate the classical turning points on these sketches.

(b) Estimate the ground state energy eigenvalue.

HINT: Note that the curvature of $\psi(x)$ near $x = 0$, *i.e.*, the second derivative of $\psi(x)$ is roughly the same for this case as for the case of an infinite one-dimensional square well of width a (for which the ground state energy is $\pi^2\hbar^2/2ma^2$). Classically the energy at $x = 0$ is all kinetic while at the turning point it is all potential.

QM-2 An electron in a hydrogen atom is in the state described by the wave function

$$\Psi(\mathbf{r}) = R_{32}(r)\left[\sqrt{\tfrac{2}{3}}\,Y_2^{-2}(\theta,\phi)\,\chi_+ + \sqrt{\tfrac{1}{3}}\,Y_2^{-1}(\theta,\phi)\,\chi_-\right],$$

where $R_{nl}(r)$ denotes the radial part of the hydrogen atom wavefunction, $Y_l^m(\theta, \phi)$ is the spherical harmonic of order lm, and χ_+ (χ_-) is the spin wavefunction for electron spin up (down). Consider the operators L^2, L_z, S^2, S_z, J^2 and J_z where **L**, **S** and **J** are the orbital angular momentum, spin angular momentum and total angular momentum, respectively. If you measured each of the corresponding quantities for this electron, what are the possible values that you might obtain (with non-zero probability)? Calculate the probability of obtaining each. Show your work and reasoning, and display your answers in a table of the form shown below.

Operator	Possible values	Probabilities (in same order)
L^2		
L_z		
S^2		
S_z		
J^2		Skip this one (Clebsch-Gordon coefficients are needed)
J_z		